DEMOGRAPHY BASICS

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OVERVIEW

- A Brief History of Demography
- Population Growth: Drivers of growth
- Demographic Indicators
  - Definition and Calculations
  - Applications in Public Health
- Calculation of District Population

DEFINITION

- Greek word 'Desmos' – Human Being
- 1855 - Guillard – Mathematical Knowledge of general movements and of physical, social, intellectual, and moral conditions of the population
- Van de Valle 1982 – The scientific study of human populations and primarily with respect to their structure and development – narrow definition

DEFINITION - DEMOGRAPY

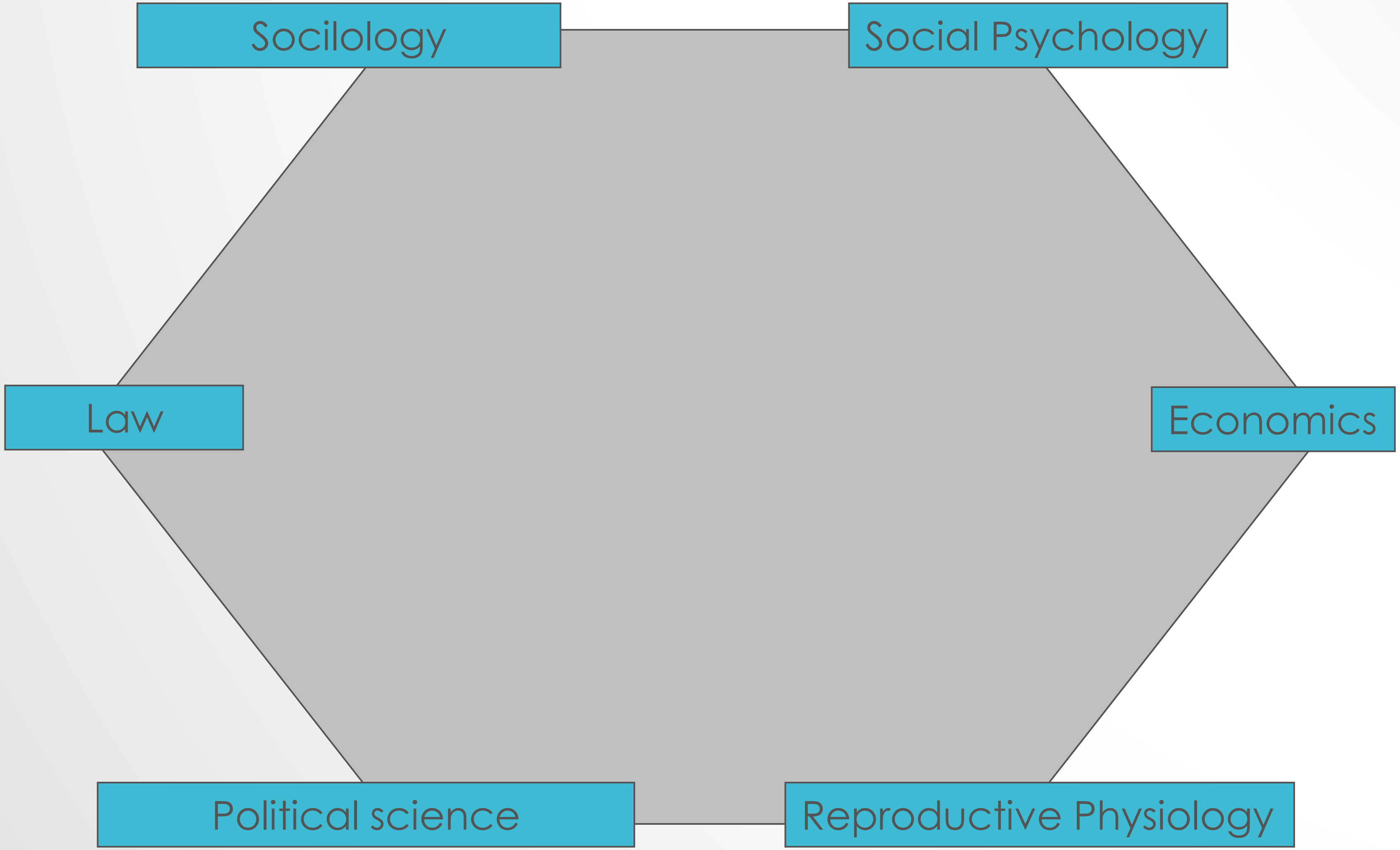
" The Scientific study of Human populations in their aggregate with regards to their size, composition or structure, spatial distributions, and developments or changes in these over time"

PLUS

The causes and consequences of these levels and changes

Population Studies

SCOPE OF DEMOGRAPHY



FORMAL DEMOGRAPHY

- Set of techniques where data collected in censuses, surveys, and vital registration systems are described, summarized and manipulated
- "The treatment of quantitative relations among demographic phenomenon in abstraction from their association with other phenomenon"

(The multilingual demographic dictionary, IUSSP)

(Van de Walle, 1982)

- Eminent Names – John Graunt, 1662 ; Edmund Halley, 1693; Johann Sussmilch, 1741-65 ; William Farr, 1841
-

FORMAL DEMOGRAPHY - HISTORY

- Stable Population Theory –
  - Leonard Euler, 1760 – Demonstrated “the constancy of the force of mortality matched by constancy of force of fertility in a closed population necessarily implied a constancy of age and sex distribution irrespective of the rate of natural increase”
  - Alfred J. Lotka, 1922 – Developed mathematical schematics of stable population

FORMAL DEMOGRAPHY – HISTORY(CONTD.)

- Kuczynski, 1935 – TFR, NRR
- Whelpton,1954 – Cohort analysis for fertility
- John Hajnal, 1953 – Singulate Mean Age at Marriage
- Coale & Trussel, 1974 - Marital Fertility model capturing the range of age patterns of human fertility

THEORIES OF FERTILITY

- Biological theories – Malthusian Theory, Salders, Double Days' Hypothesis, Pearl and Reed hypothesis, Herbert Spencer's Theory, Jouse de Castro's hypothesis, Robert Ardreys hypothesis etc
- Sociocultural & Economic Theories – Marxian Theory of surplus population, Desmartes hypothesis, Frank Felter's, Henry George's, Harvey Leibenstein's, Becker Model, Easerliner's hypotheis etc

THEORIES OF FERTILITY

- Malthusian Thoery – Thomas Malthus, 1798 – An Essay on Principle of Population
- Three Postulates
  - Passion between sexes is inevitable and universal – Population increases at a rapid, geometric rate
  - Food production is limited – grows at arithmatical rate – Population cannot increase beyond a level that can be sustained by given level of food availability
  - If it outstrips the means of substinence, positive checks will apply
- Checks – Natural - famines, wars, epidemics,etc or Prudent checks – celibacy, natural family planning methods

BASIC DEMOGRAPHIC MEASURES

DEMOGRAPHIC EQUATION

$$P_2 = P_1 + B - D + I - E$$

- Where, P_2 & P_1 – Populations at two different times

B = Births
D = Deaths
} Natural Increase

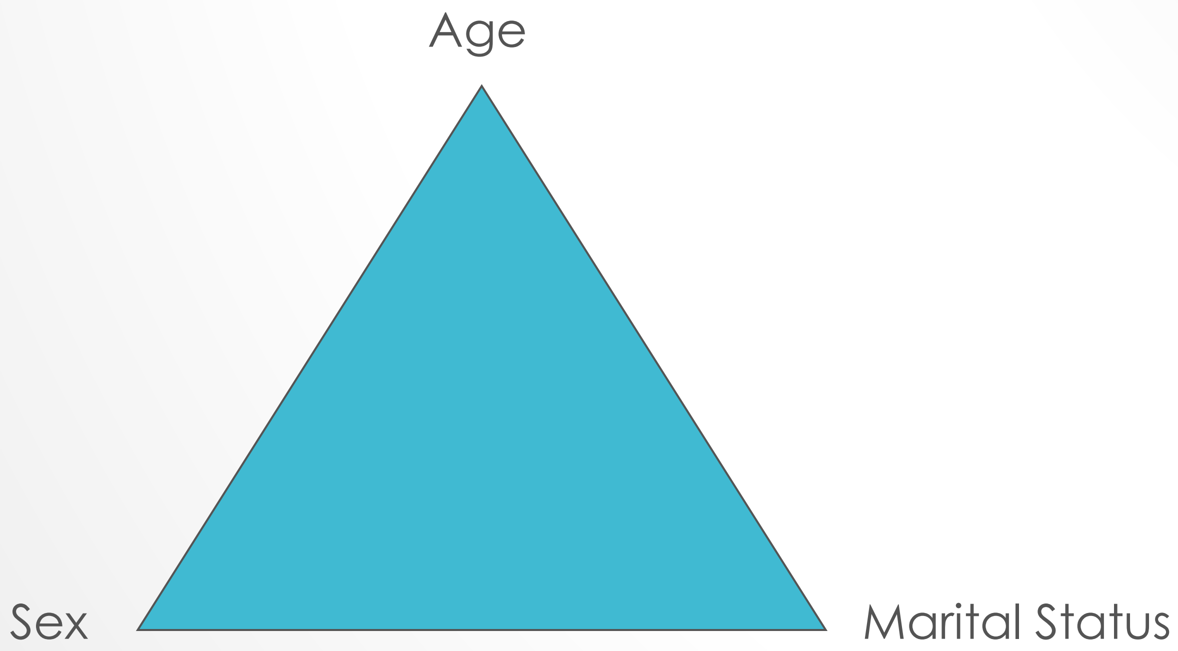
I = Immigrants
E = Emigrants
} Net Migration

DEMOGRAPHIC PROCESSES

- Fertility or births
- Mortality or deaths, and
- Migration

Contribute to changes in size and structure of populations

BASIC DEMOGRAPHIC VARIABLES



MID-YEAR POPULATION

- Demographic measures often use *mid-year* population
- Mid-year population is NOT the same as population at risk
  - Population is risk is usually a sub-set of the *initial* population
    - not a sub-set of *mid-year population*

$P_{mid} = (P_{start} + P_{end}) / 2$

$= (Po + Pn)/2$

$= Po + \frac{1}{2}(Pn - Po)$

POPULATION SIZE AT ANY GIVEN POINT IN A YEAR

Average Rate of change over the period

$$Po + t\,.\left(\frac{1}{n}(Pn - Po)\right)$$

Where,

n = Total number of periods in which the year has been divided

t = The number of time periods at the end of which the population size is required

Eg. Population at 28st Feb:

Divided year in 365 days, and

Calculate population after 31 + 28 days (Jan 31 days, Feb 28 days)

$$Po + (31 + 28)\,.\left(\frac{1}{365}(Pn - Po)\right)$$

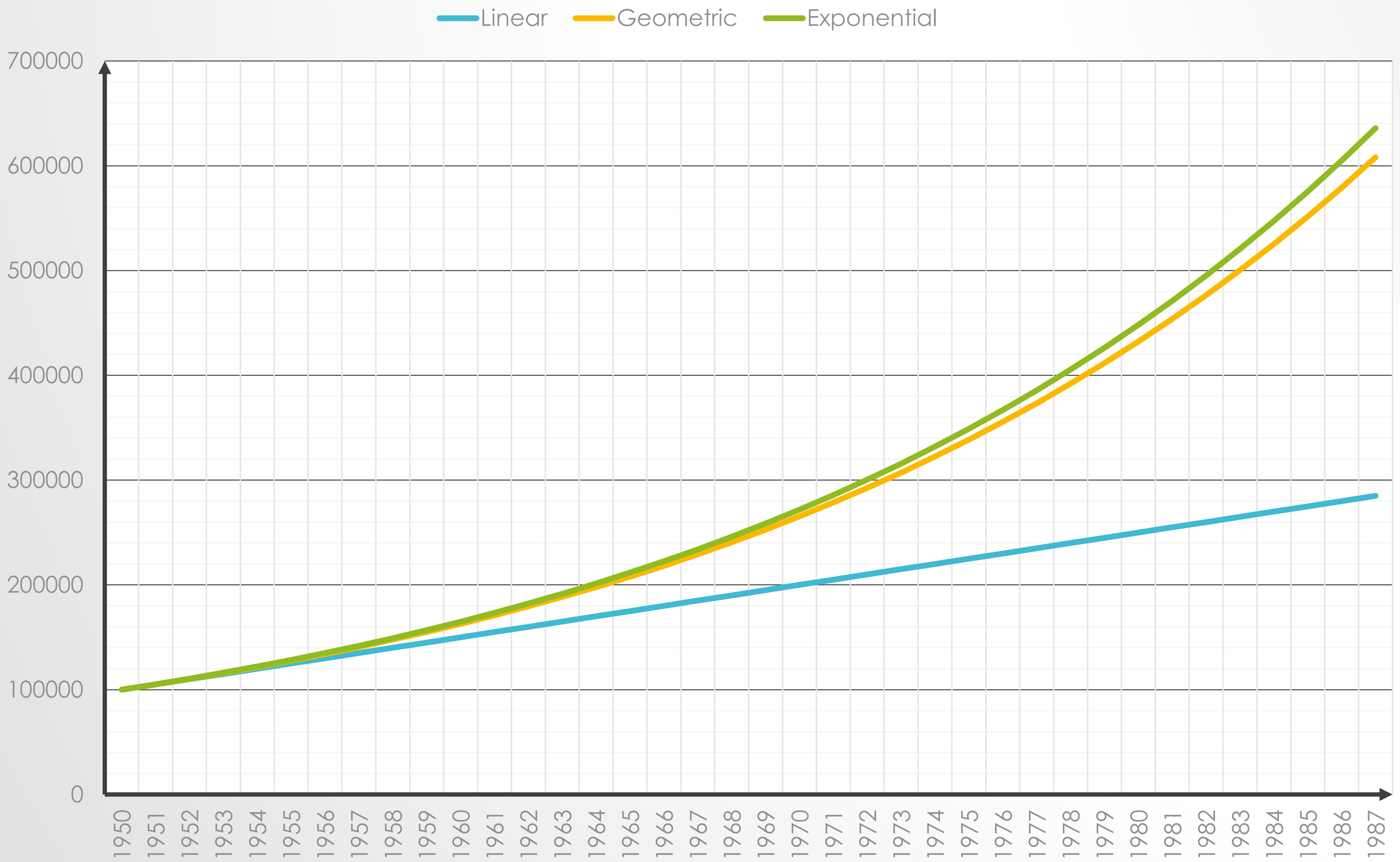
BASIC MEASURES OF POPULATION CHANGE

- Crude Birth Rate (CBR) = $\frac{Births\ in\ a\ year}{Mid\ year\ population} \times 1000$
- Crude Death Rate (CDR) = $\frac{Deaths\ \ in\ a\ year}{Mid\ year\ population} \times 1000$
- Rate of Natural Increase = CBR – CDR

$$= \frac{Births - deaths\ \ in\ a\ year}{Mid\ year\ population} \times 1000$$

- Rate of Net Migration = $\frac{net\ migration\ \ in\ a\ year}{Mid\ year\ population} \times 1000$

Population Growth Patterns



MEASURES OF POPULATION CHANGE

	Linear Growth	Geometric Growth (Compound growth)	Exponential Growth
End of Period Population P_n	$Pn = Po + annual\ absolute\ growth\ .\ n$	$Pn = Po\ .\ (1+r)^n$ $\log Pn = \log Po + \log(1+r) \times n$	$Pn = Po\ .\ e^{r\ .\ n}$ $\ln Pn = \ln Po + rn$
Growth Rate r **(Expressed as fraction)**	$r = \left(\frac{Pn - Po}{n}\right) \div Po$	$r = \sqrt[n]{\frac{Pn}{Po}} - 1$ Geometric Growth Rate	$r = \frac{\ln\left(\frac{Pn}{Po}\right)}{n}$ Exponential Growth Rate
Doubling Time	$\frac{Po}{annual\ absolute\ growth}$	$\frac{\log 2}{\log(1+r)}$	$\frac{\ln 2}{r} \equiv \frac{0.6931}{r}$ (where r is a fraction)

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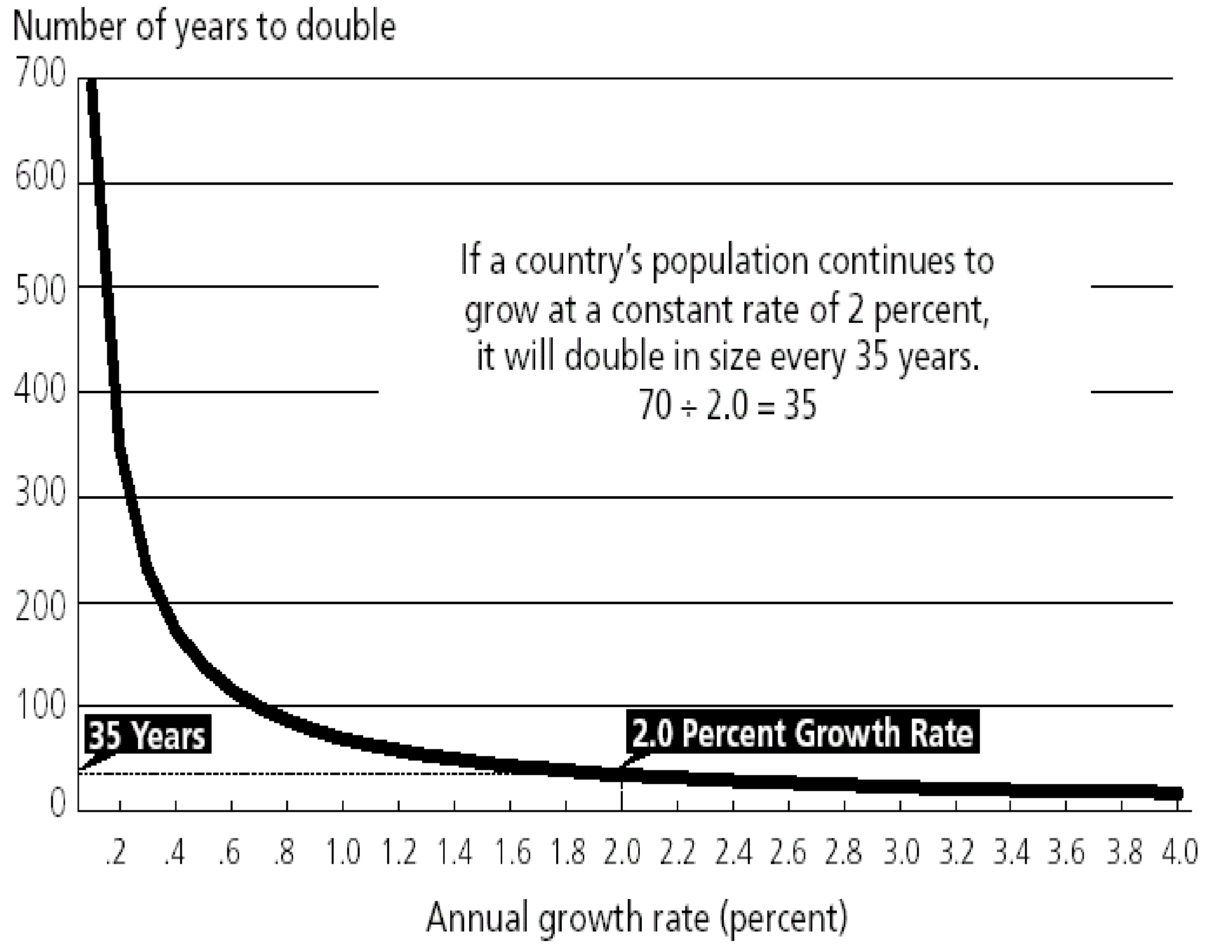
Based on exponential growth

Doubling Time

The doubling time of a population is simply the number of years it would take for a population to double in size if the present rate of growth remained unchanged. Used for many years, its primary purpose has been to emphasize just how quickly populations can grow, doubling their numbers geometrically.



LAW OF 70

Mathematical Relationship is

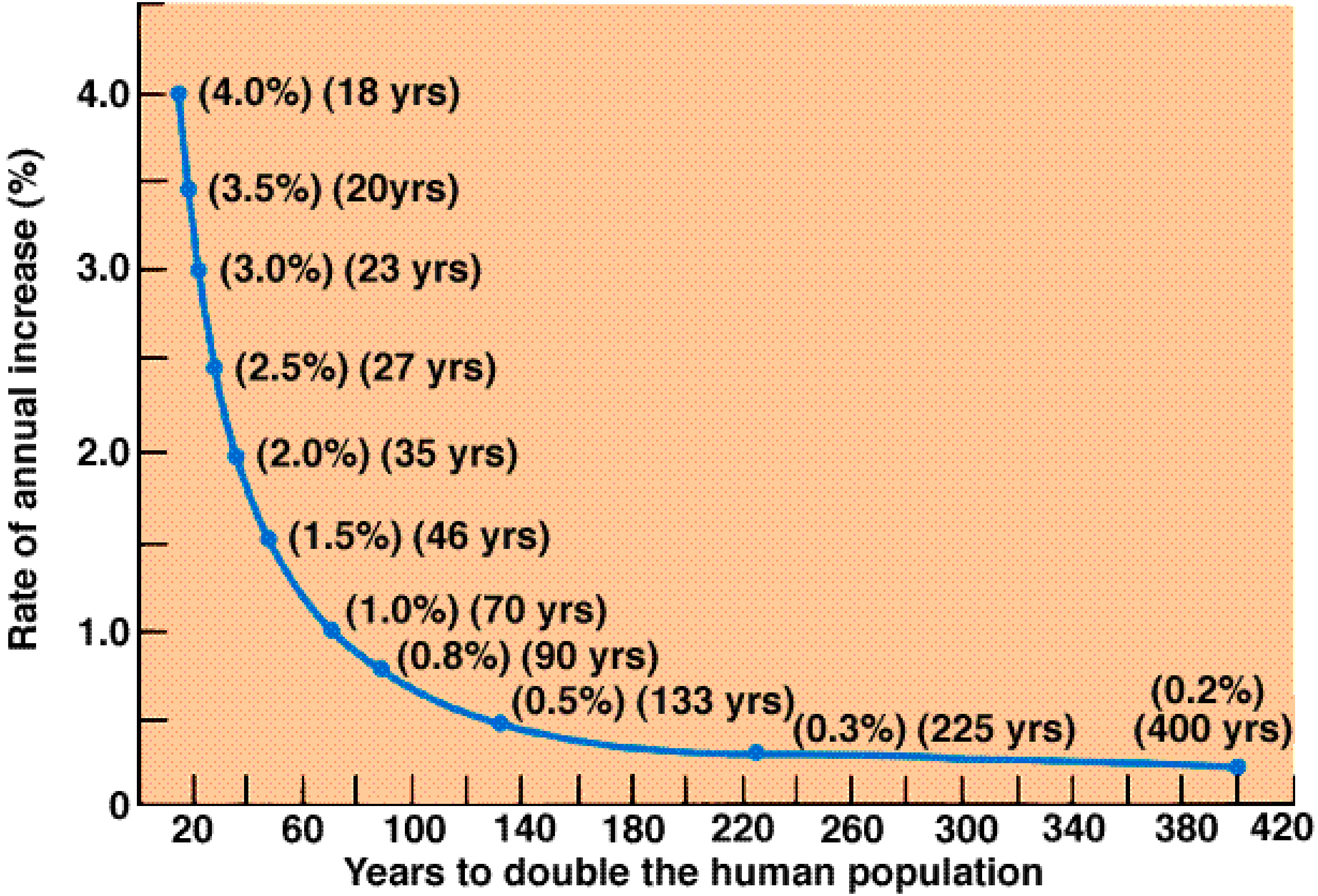
$$rt = 69.31$$

Where,

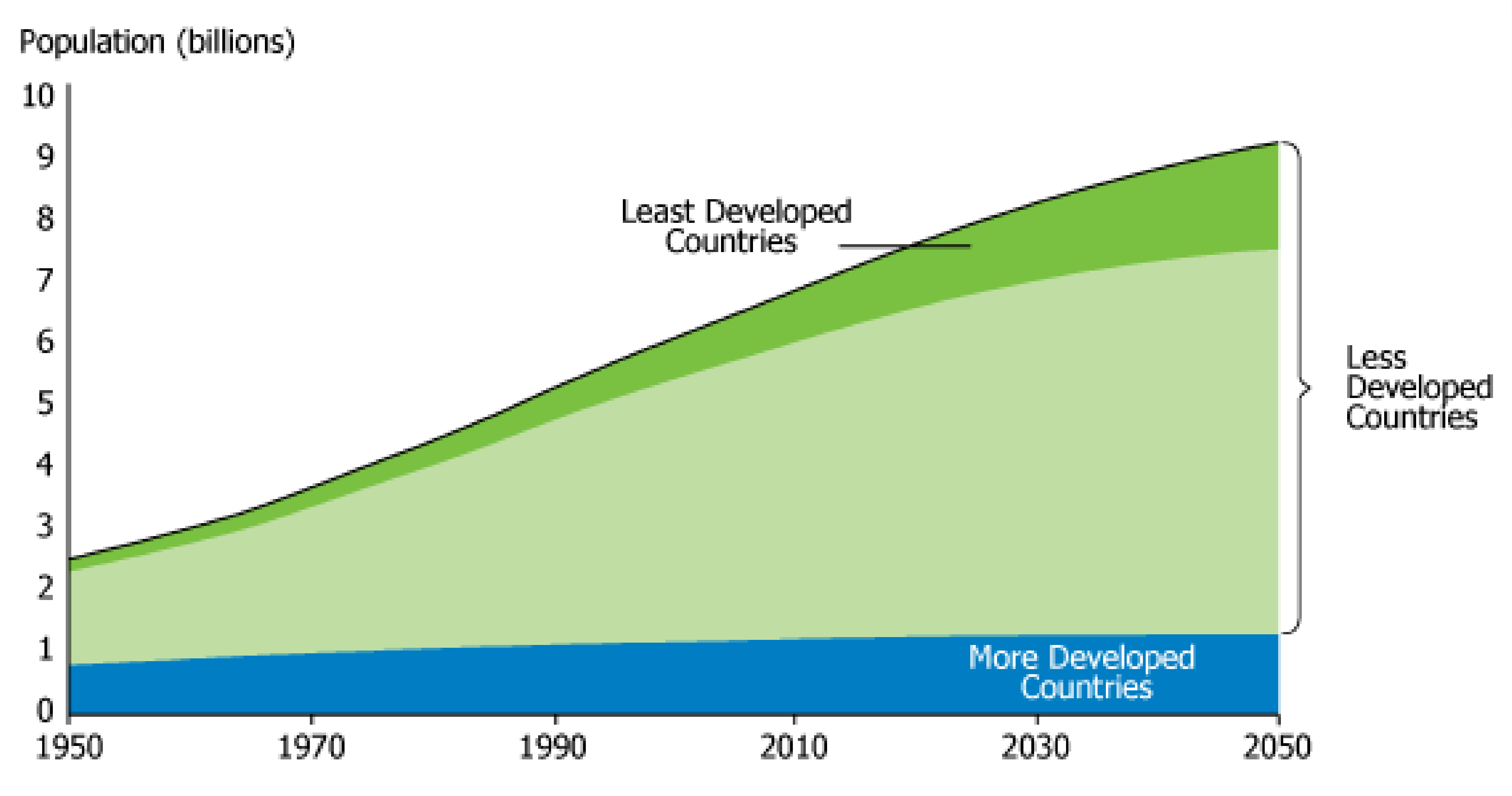
r = Annual Growth rate in percentage

t = Time in years

Doubling Time for the Human Population

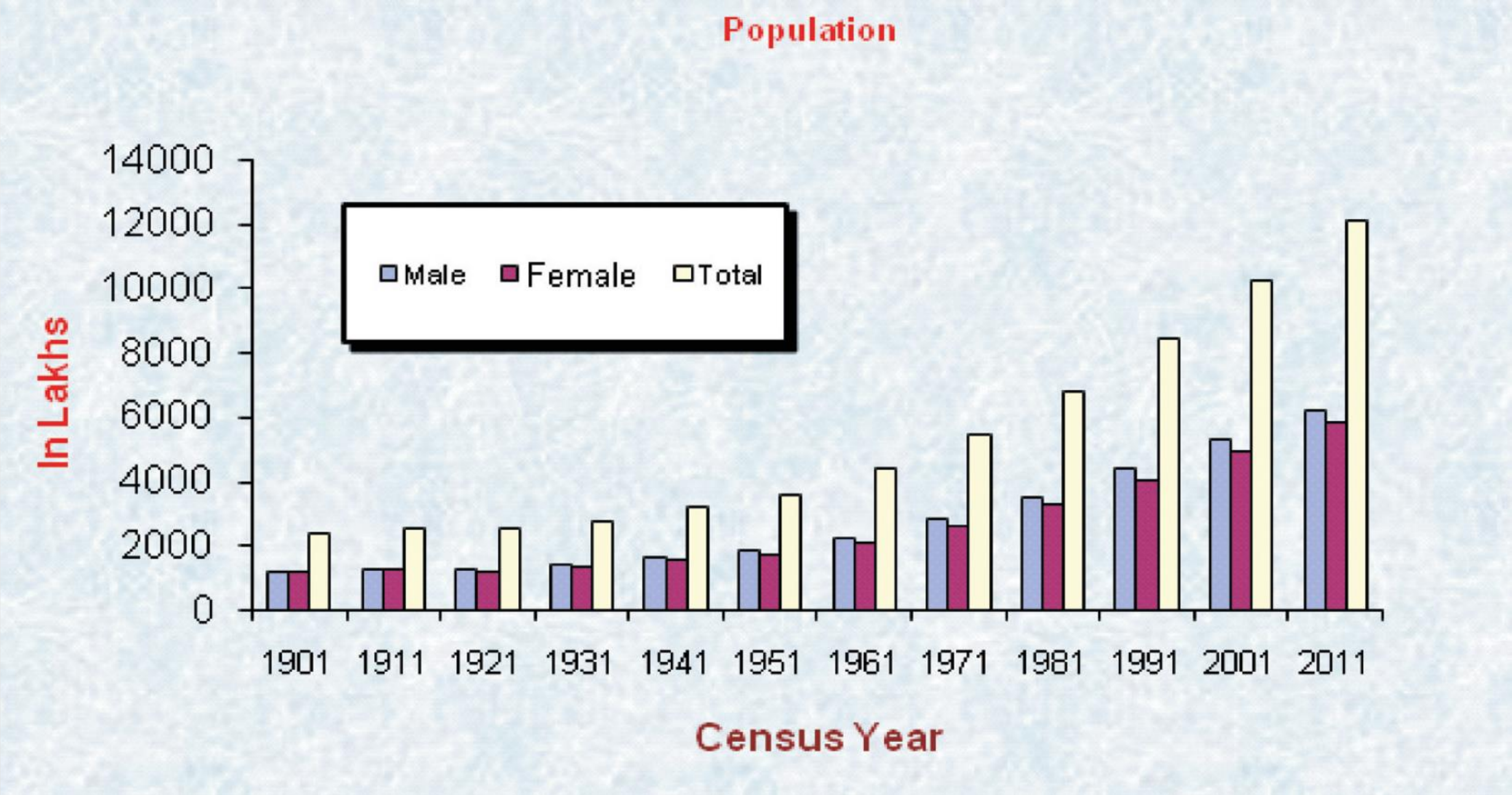


GROWTH OF WORLD POPULATION



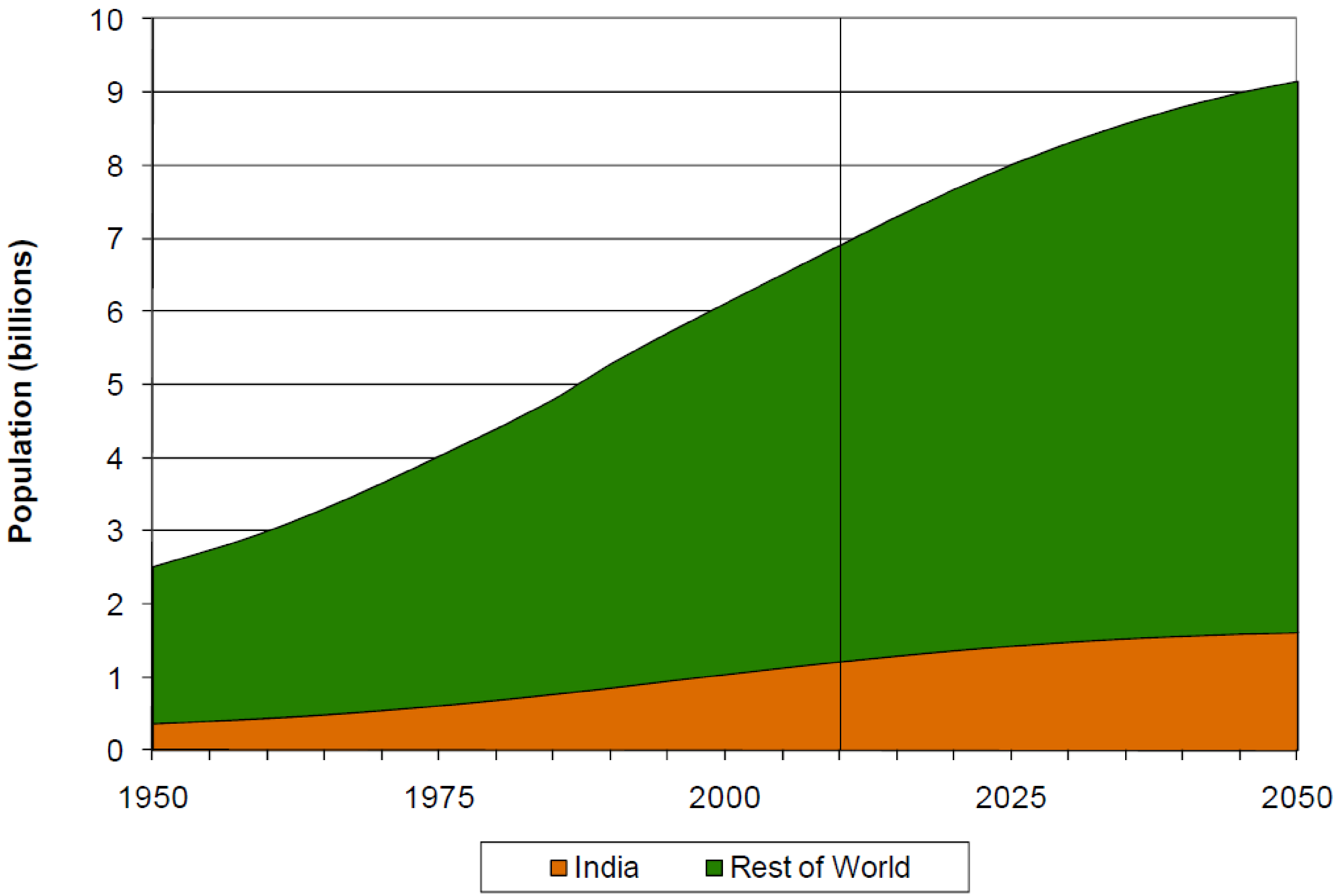
Source: United Nations Population Division, *World Population Prospects: The 2010 Revision,* medium variant (2011).

INDIA'S POPULATION GROWTH



National Health Profile 2011

India's share of world population



Source: United Nations (2009).

AGE AND SEX COMPOSITION

AGE

- The most important variable in demographic analysis
  - Age at Last Birthday generally used
  - Most difficult characteristic to be ascertained in view of high illiteracy rates in India
- Mean Age of a Population - weighed average of the population distribution by age

AGE STRUCTURE

- Influences future demographic events
- Social and economic conditions vary with age
- Knowing age structure of utmost importance to a planner

National Health Profile 2011

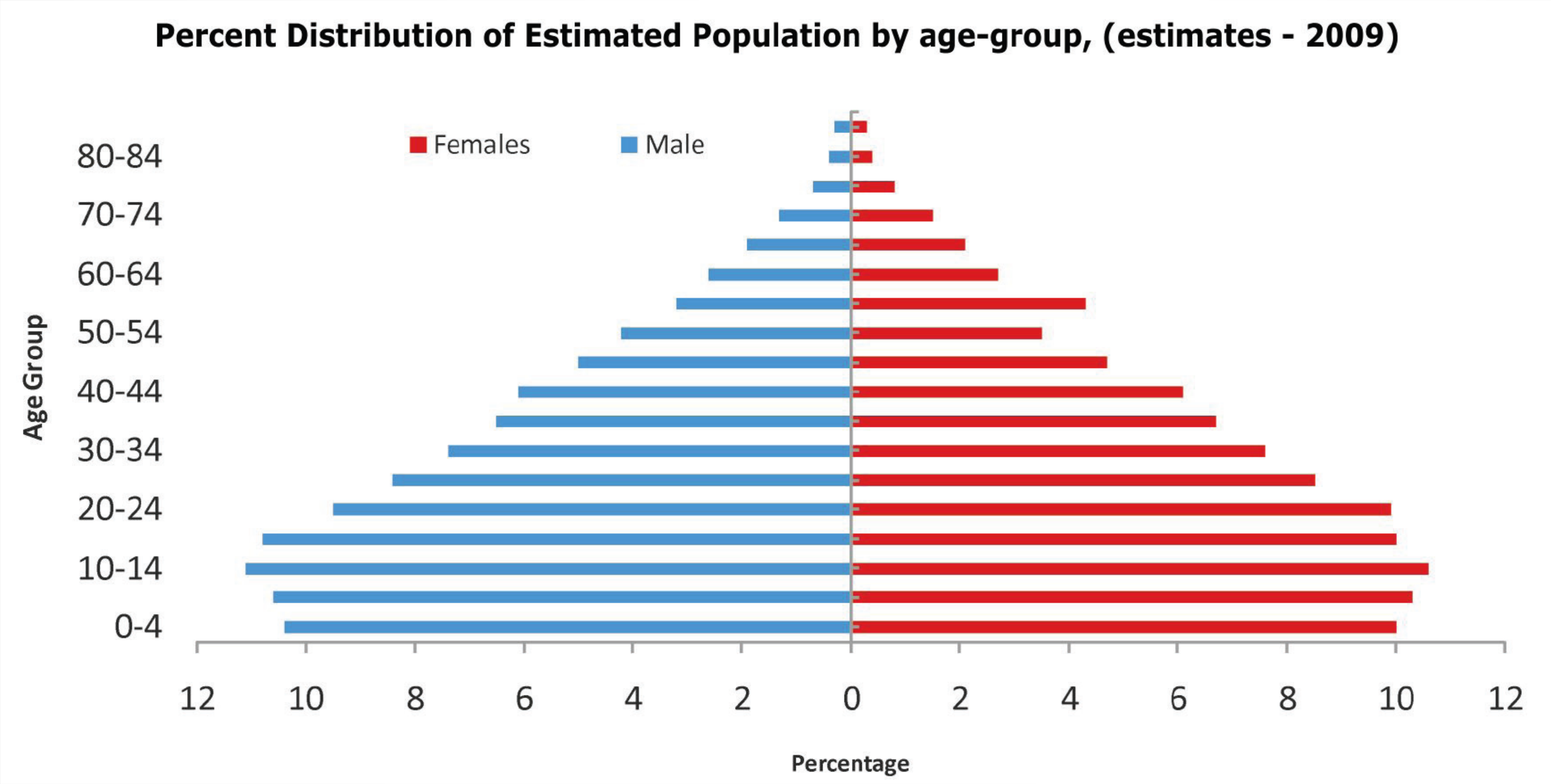
POPULATION PYRAMID

- Age and Sex composition of a population
- Male population kept on left hand side
- Female population kept on right hand side
- Pyramid starts with lower ages
- Advisable to drop the open ended age group at oldest ages
- Population can be in Actual figures or Percentage

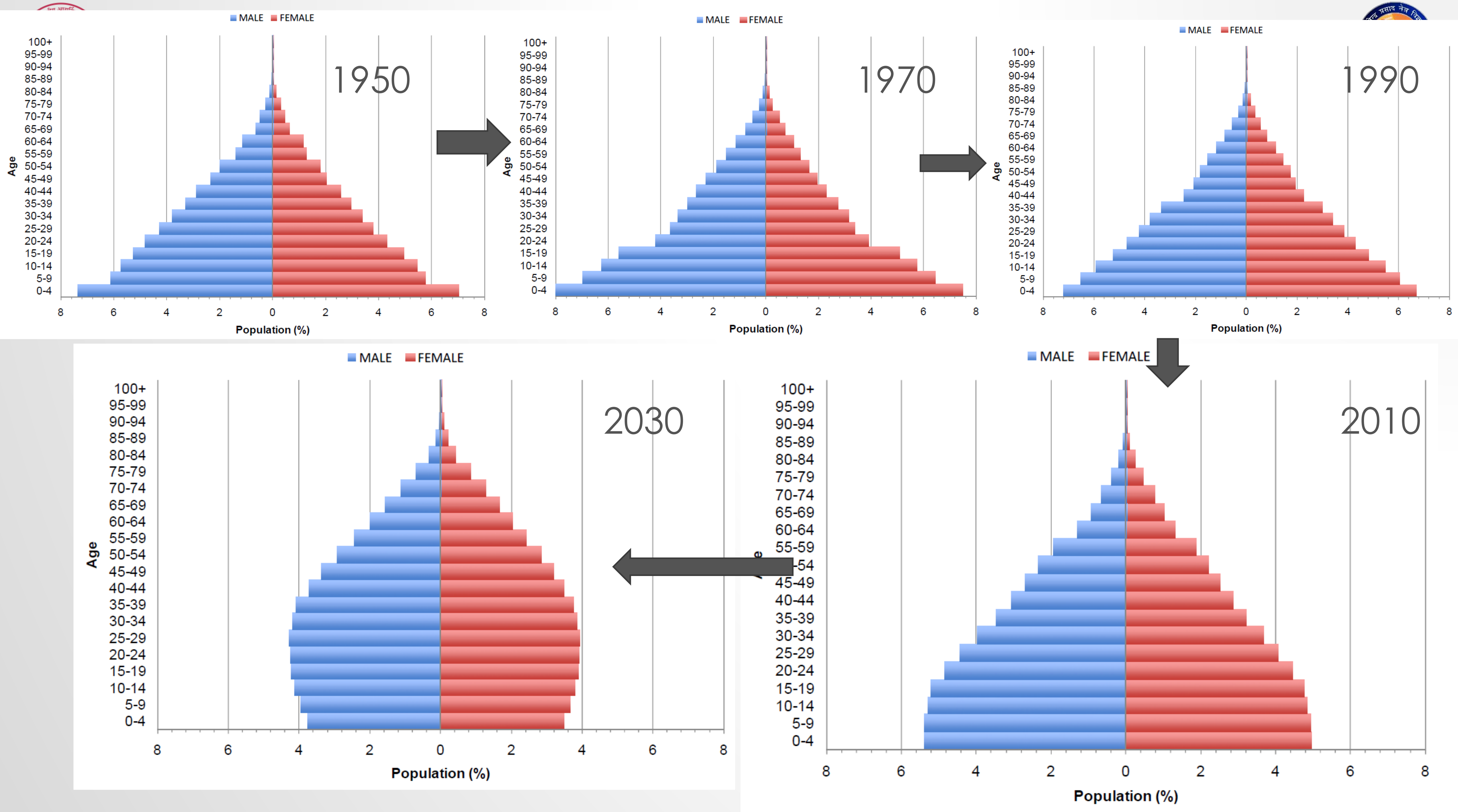
AGE STRUCTURE

- Affected by process of
  - Births – most important determinant
  - Deaths –
    - infant deaths
    - adult deaths – distributed uniformly over all ages
  - Migration – can cause changes in structure as age selective process – young adults

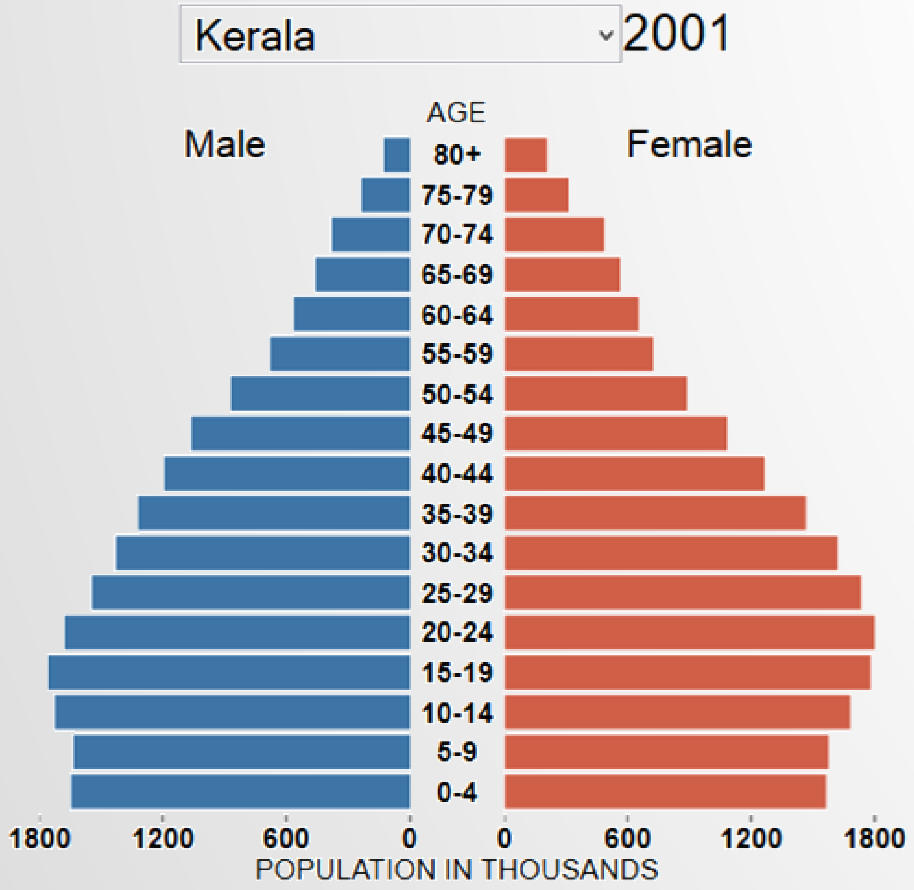
Population ages when fertility declines resulting in fewer births.

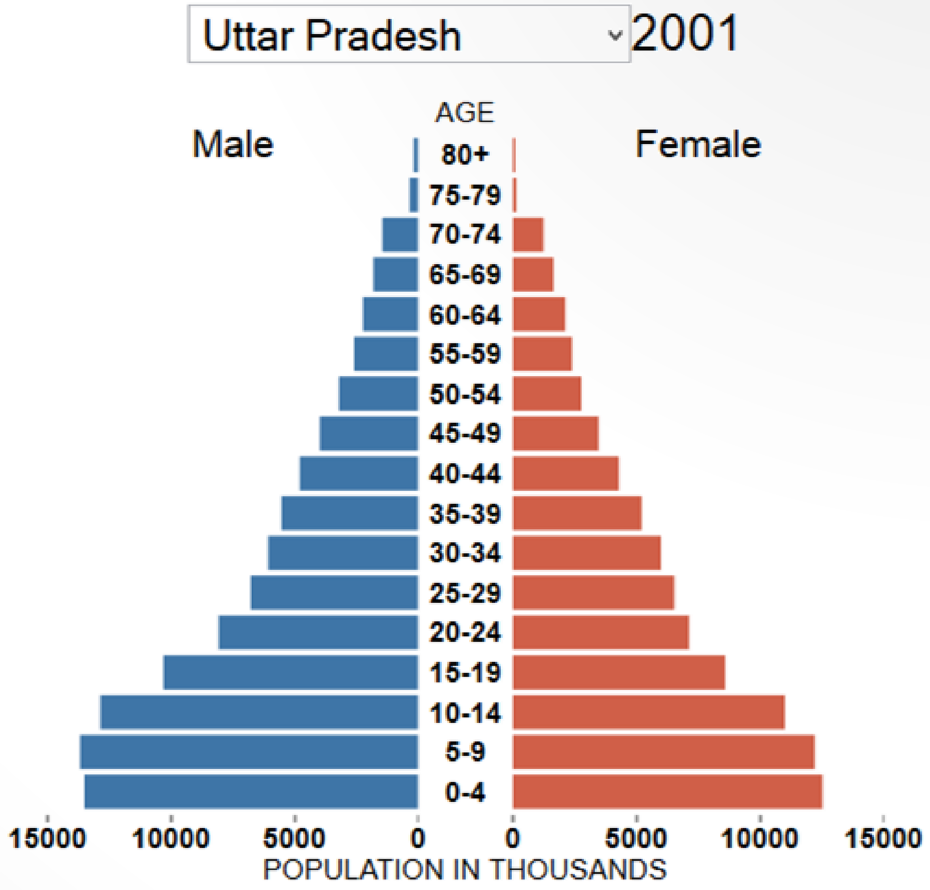


National Health Profile 2011



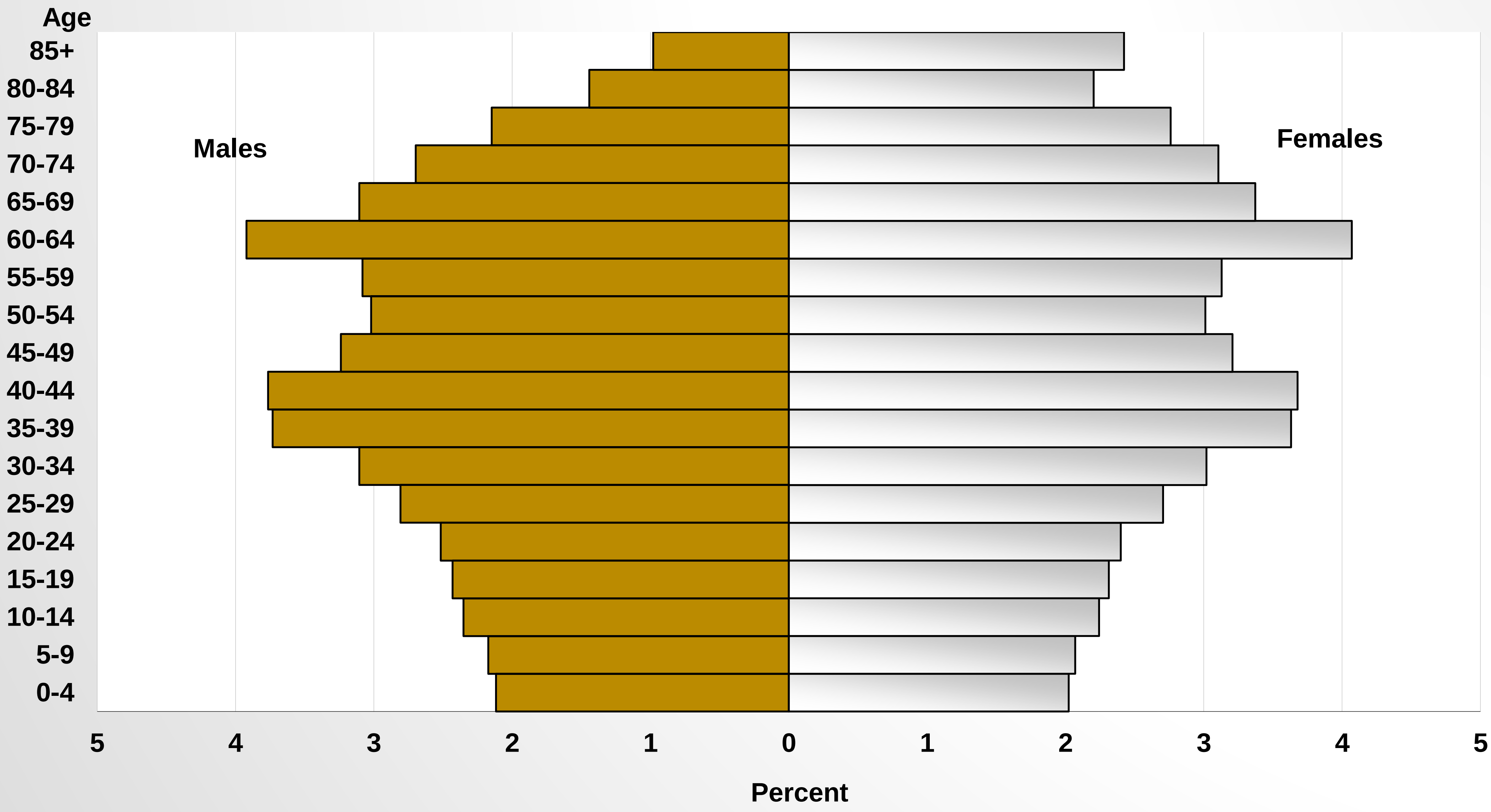
MALE
FEMALE
1950
1970
1990
2010
2030
Age
100+
95-99
90-94
85-89
80-84
75-79
70-74
65-69
60-64
55-59
50-54
45-49
40-44
35-39
30-34
25-29
20-24
15-19
10-14
5-9
0-4
8
6
4
2
0
2
4
6
8
Population (%)





http://www.sciencemag.org/site/special/population/1207969-james-f4.xhtml?utm_content=short&utm_medium=magazine&utm_campaign=science&utm_source=shortener

Japan by Age and Sex, November 1, 2012



Source: Japan Statistics Bureau

MEASURES OF AGE-SEX COMPOSITION

- Sex ratio
  - Males per 100 females: Internationally
  - Females per 1000 males: used in India

- Dependency ratio   Ratio $P_{0\text{-}14} + P_{65+}$ to $P_{15\text{-}64}$
  - Child dependency ratio
  - Aged dependency ratio
  - Economic dependency ratio

- Ageing Index   Ratio of P_{65+} to $P_{0\text{-}14}$
- Caretaker ratio   Ratio of P_{80+} to Females in $P_{50\text{-}64}$

SEX COMPOSITION

- Sex Ratio –

$$\frac{\text{Number of female in a population at specified time}}{\text{Number of males in a population at specified time}} \times 1000$$

- Age Specific ratios can also be calculated
- Important as demographic events such as fertility are sex specific
- Also differences between death rates of sexes

MEASURES OF CENTRAL AGE

- Mean Age = sum of all ages / population

- Median Age = $l + \left(\frac{N}{2} - F\right) \times \left(\frac{i}{f}\right)$

Where:

l = lower limit of the class containing the middle case

N = total population

F = cumulative frequency up to the age group containing the middle case

f = frequency of the class containing the middle class

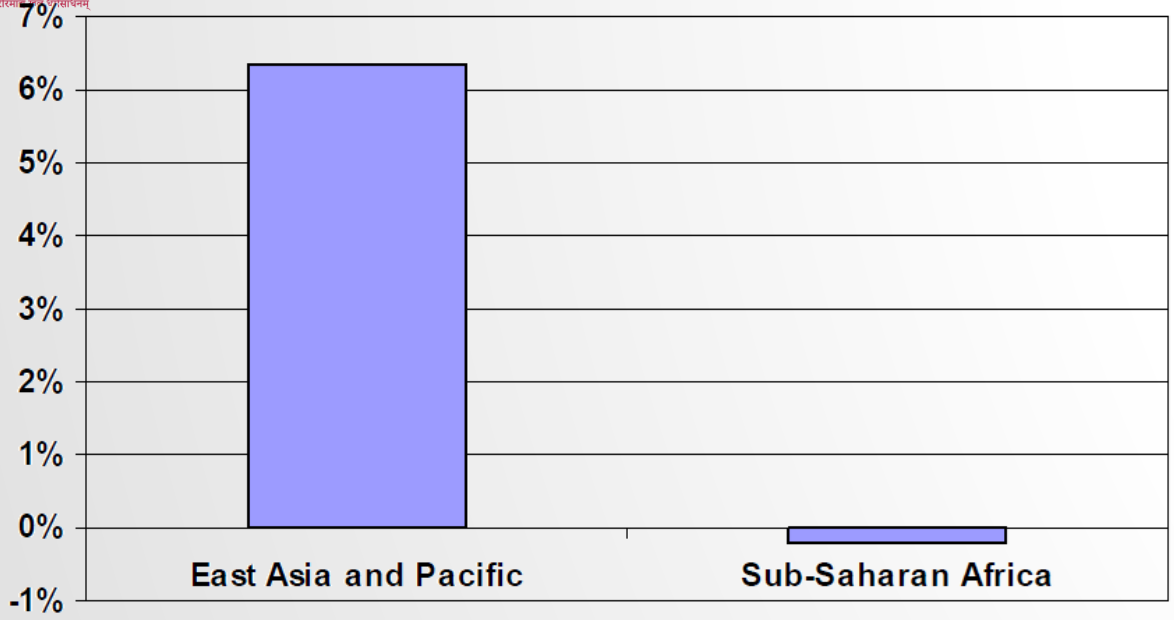
i = the size of the class interval containing the middle case

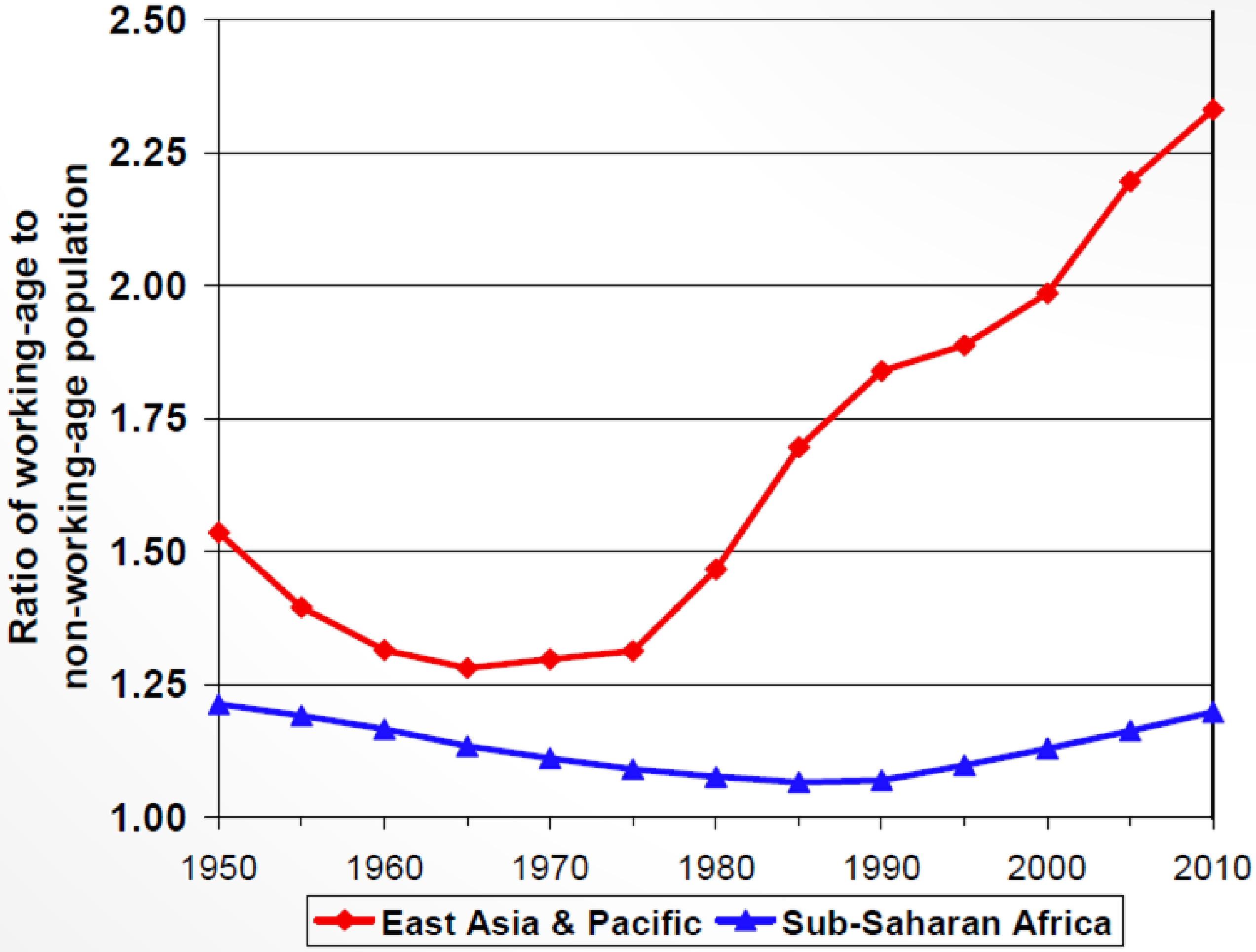
DEPENDENCY RATIO

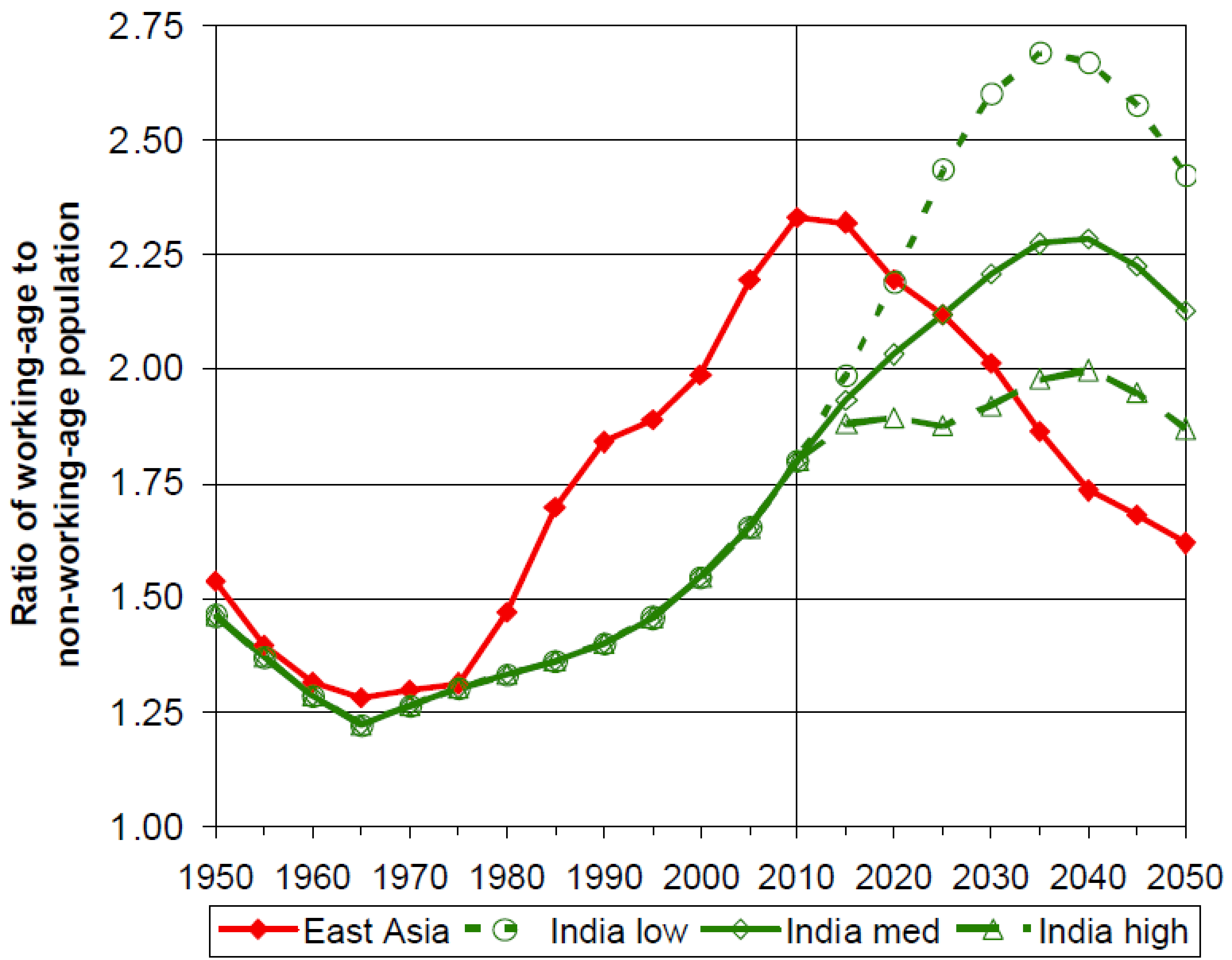
$$\frac{P_{0-14} + P_{65+}}{P_{15-64}} \times 100$$

- Measure of economic dependence in a population
- Separate Child Dependency Ratio and Old Age Dependency Ratio can also be calculated

Average annual growth rate of GDP per capita, 1975-2005





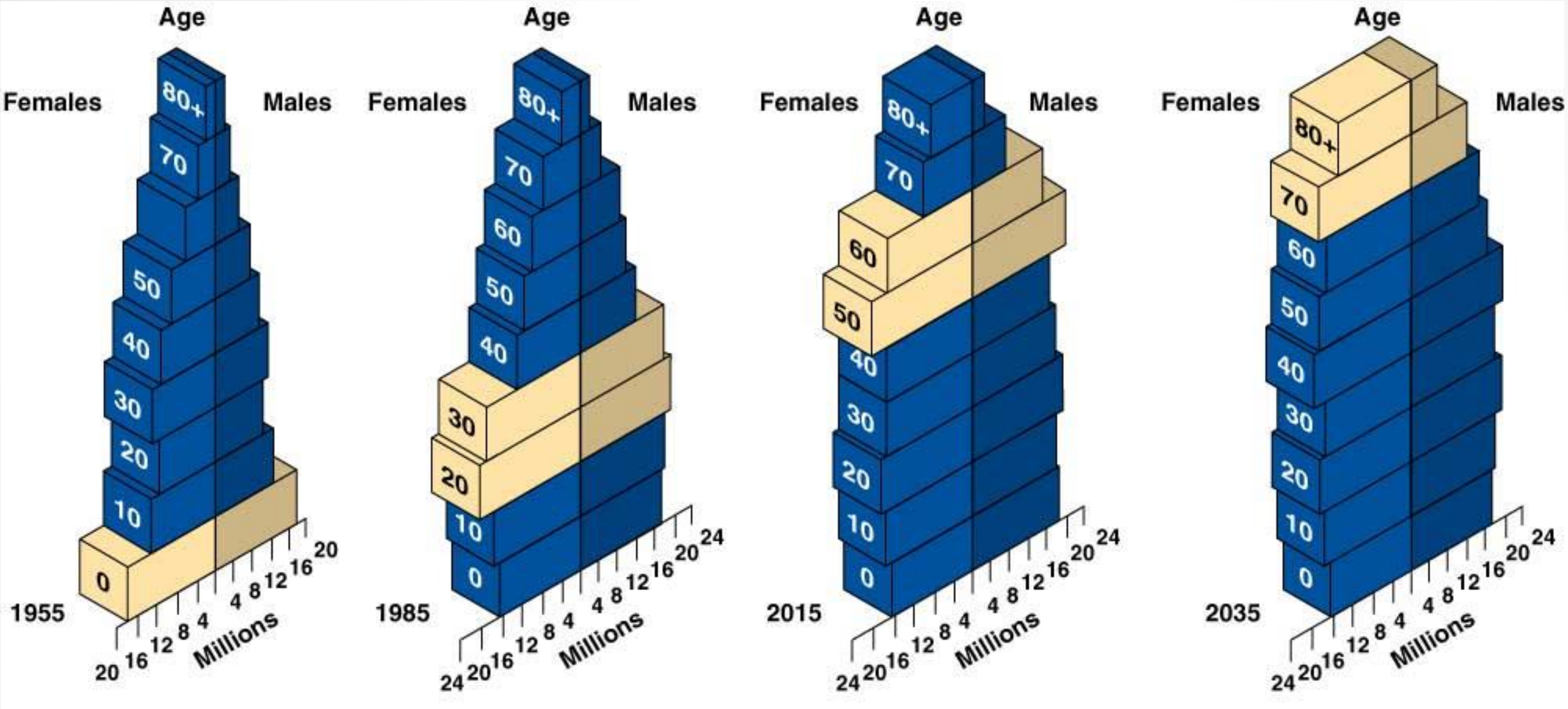


Ratio of working-age to non-working-age population
2.75
2.50
2.25
2.00
1.75
1.50
1.25
1.00
1950
1960
1970
1980
1990
2000
2010
2020
2030
2040
2050
East Asia
India low
India med
India high

AGE STRUCTURE

- Baby boomers - half of U.S. population; use most of goods and services; make political and economic decision
- baby-bust generation - born since 1965; may have to pay more income, health care and social security to support retired baby boomers; but face less job competition
- Better health may --> later retirement of baby boomers --> keep high-salary jobs

Tracking the baby-boom generation in the United States

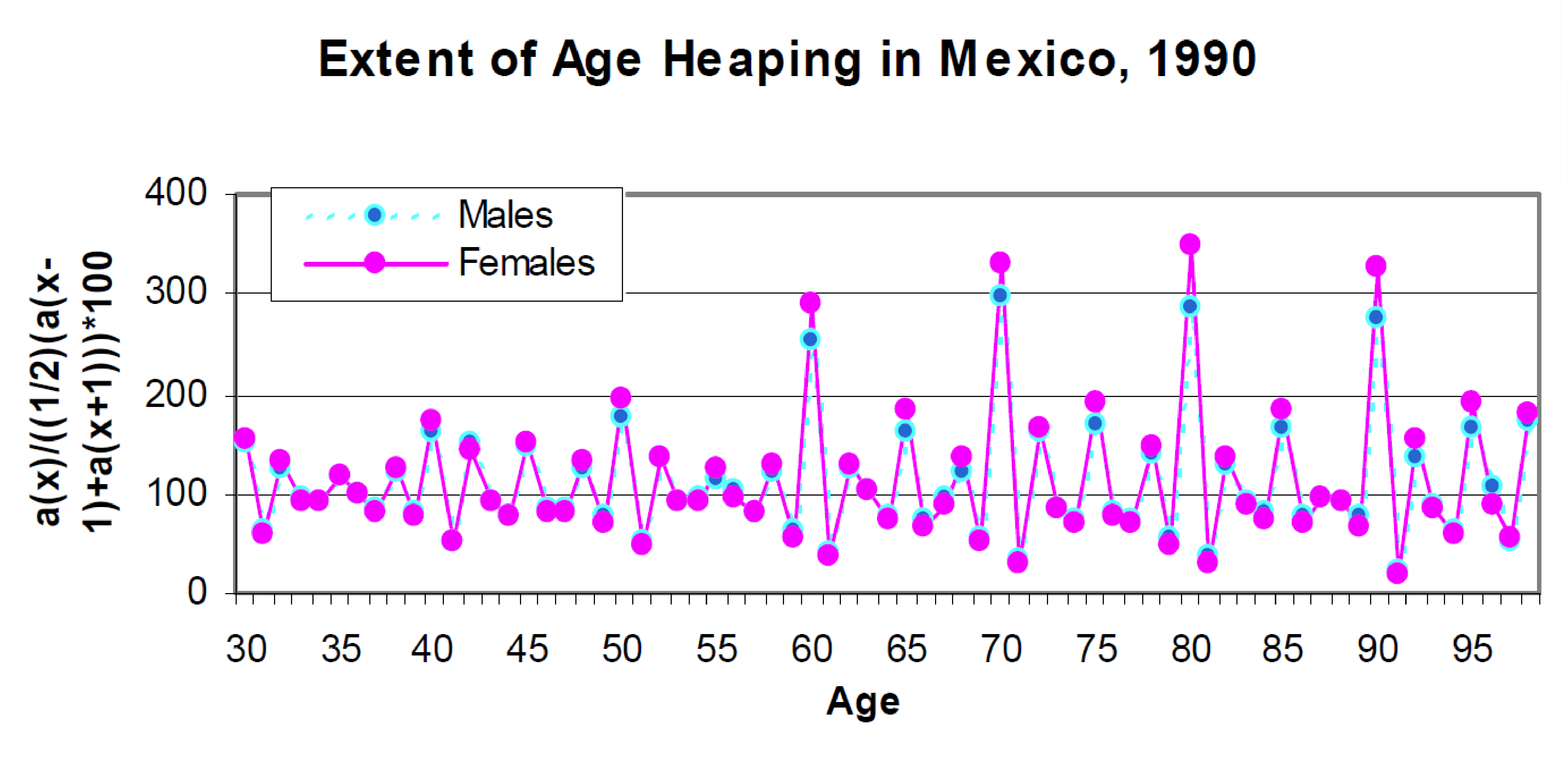


DEMOGRAPHIC DIVIDEND

- Accelerated economic growth that may result from a decline in a country's birth and death rates and the subsequent change in the age structure of the population.
- With fewer births each year, a country's young dependent population declines in relation to the working-age population.
- With fewer people to support, a country has a window of opportunity for rapid economic growth if the right social and economic policies are developed and investments made.

ERRORS IN AGE DATA

- Centenarians - Those close to 100 years tend to overestimate their age
- Understatement - Women tend to understate their age
- Overstatement: Mothers tend to round up the age of their children
- Heaping/Digit preference
  - People tend to report certain ages at the expense of others
  - Can occur at any digit but happens most often with 0 and 5
- Coverage—Missed or counted twice
  - There is a tendency to miss the people in certain age groups (e.g. young men)
  - Some people are counted twice


Extent of Age Heaping in Mexico, 1990
Males
Females
400
300
200
100
0
a(x)/((1/2)(a(x-1)+a(x+1)))*100
30
35
40
45
50
55
60
65
70
75
80
85
90
95
Age

AGE RATIO

- Age ratio for an age group – the number of persons in that age group divided by the number obtained from adding one half of the numbers in the preceding and the next age group
- Eg. For a five year age group

$$\frac{{}_5P_a}{0.5({}_5P_{a-5} + {}_5P_{a+5})} \times 100$$

- Used primarily for checking net age misreporting

COMPARING POPULATIONS

COMPARING POPULATIONS

- Comparing Rates
  - $CDR_{CountryA}$ compared with $CDR_{CountryB}$
  - Age specific death rates of Males vs Age-specific death rates of females
  - Sometimes, Ratio of country A vs Couny B are calculated: Rate Ratios
    - How these differ from rate-ratios calculated in epidemiology ?
      - (Hint. denominator)
- Reasons for variations in across populations
  - Demographic (age-sex) composition of the populations different
  - Demographic (age-sex) composition of the populations similar but age-sex specific rates vary
  - Quality of data varying across populations: under-reporting, mis-reporting,

STANDARDIZATION

- Enables appropriate comparisons after accounting for differences in
  - Demographic (age-sex) composition
  - Age-sex specific rates
- Two Types
  - Direct Standardization
  - Indirect Standardization
- History: http://www.who.int/healthinfo/paper31.pdf

DIRECT STANDARDIZATION

- Choose a *standard population*
  - What is the standard population ??
    - One of the populations: Country A / B / C
    - Sum of populations: Country A + B + C…
    - International standard population
    - Any other population..
  - Choose an Informative standard population

Standard Population Distribution (percent)

Age Group	Segi ("world") standard	Scandinavian ("European") standard	The New WHO World Standard
0- 4	12.00	8.00	8.86
5- 9	10.00	7.00	8.69
10-14	9.00	7.00	8.60
15-19	9.00	7.00	8.47
20-24	8.00	7.00	8.22
25-29	8.00	7.00	7.93
30-34	6.00	7.00	7.61
35-39	6.00	7.00	7.15
40-44	6.00	7.00	6.59
45-49	6.00	7.00	6.04
50-54	5.00	7.00	5.37
55-59	4.00	6.00	4.55
60-64	4.00	5.00	3.72
65-69	3.00	4.00	2.96
70+	2.00	3.00	2.21
75-79	1.00	2.00	1.52
80-84	0.50	1.00	0.91
85+	0.50	1.00	0.63
Total	100.00	100.00	100.00
	Segi M. Cancer mortality for selected sites in 24 countries (1950-57). Department of Public Health, Tohoku University of Medicine, Sendai, Japan. **1960**.	Doll R, & Cook P. Summarizing indices for comparison of cancer incidence data. Int J Cancer 2:269-79, **1967**.	Ahmad OA, Boschi-Pinto C, Lopez AD, Murray CJL, Lozano R, Inoue M. Age standardization of rates: A new WHO standard. GPE Discussion Paper Series: No.31 EIP/GPE/EBD. World Health Organization **2001**

DIRECT STANDARDIZATION

- Calculate age specific death rates (ASDRs) for each population
  - Do not round off the rates too much
- Calculate the Expected number of deaths in each age-group in the standard population based on ASDRs of first population
- Calculate the TOTAL Expected number of deaths across ALL age-group in the standard population based on ASDRs of first population
- Calculate the Age-standardized Crude Death Rate for first population
- Repeat the process for each population being compared
- Compare the Age-standardized Crude Death Rates of various populations

Country	Australia		Australia	
Year	1921		1981	
	Population	Deaths	Population	Deaths
0- 4	600206	11980	1111945	2772
5- 9	595758	1168	1249941	327
10-14	529040	795	1295018	319
15-19	464217	985	1259029	1075
20-24	454027	1385	1247783	1260
25-29	462449	1734	1184149	1089
30-34	449322	1951	1192232	1076
35-39	387132	2168	977233	1238
40-44	331518	2151	822701	1674
45-49	280477	2376	723139	2621
50-54	256066	2899	757544	4459
55-59	215914	3488	726485	6942
60-64	169342	4116	599732	8734
65-69	104998	3877	524885	12286
70+	135268	12616	904514	61192
Total	**5435734**	**53689**	**14576330**	**107064**

Crude death rate 9.88 7.34

CALCULATE AGE SPECIFIC DEATH RATES

Country	Australia			Australia		
Year	1921			1981		
	Population	Deaths	**ASDR**	Population	Deaths	**ASDR**
0- 4	600206	11980	19.95991	1111945	2772	2.492918
5- 9	595758	1168	1.960496	1249941	327	0.261942
10-14	529040	795	1.502897	1295018	319	0.246041
15-19	464217	985	2.120853	1259029	1075	0.853939
20-24	454027	1385	3.050719	1247783	1260	1.009603
25-29	462449	1734	3.7505	1184149	1089	0.919698
30-34	449322	1951	4.341128	1192232	1076	0.90252
35-39	387132	2168	5.599584	977233	1238	1.267231
40-44	331518	2151	6.487855	822701	1674	2.034699
45-49	280477	2376	8.469647	723139	2621	3.624794
50-54	256066	2899	11.32285	757544	4459	5.885668
55-59	215914	3488	16.15607	726485	6942	9.555927
60-64	169342	4116	24.30529	599732	8734	14.56282
65-69	104998	3877	36.92904	524885	12286	23.40685
70+	135268	12616	93.26414	904514	61192	67.65162
Total	**5435734**	**53689**		**14576330**	**107064**	

CHOSE A STANDARD POPULATION, CALCULATE EXPECTED DEATHS

Country	Australia			Australia			
				Standard			Expected deaths in standard population based on ASDRs of 1921
Year	1921			1981			
	Population	Deaths	**ASDR**	Population	Deaths	**ASDR**	
0- 4	600206	11980	19.95991	1111945	2772	2.492918	22194.32
5- 9	595758	1168	1.960496	1249941	327	0.261942	2450.504
10-14	529040	795	1.502897	1295018	319	0.246041	1946.278
15-19	464217	985	2.120853	1259029	1075	0.853939	2670.215
20-24	454027	1385	3.050719	1247783	1260	1.009603	3806.636
25-29	462449	1734	3.7505	1184149	1089	0.919698	4441.15
30-34	449322	1951	4.341128	1192232	1076	0.90252	5175.632
35-39	387132	2168	5.599584	977233	1238	1.267231	5472.099
40-44	331518	2151	6.487855	822701	1674	2.034699	5337.565
45-49	280477	2376	8.469647	723139	2621	3.624794	6124.732
50-54	256066	2899	11.32285	757544	4459	5.885668	8577.556
55-59	215914	3488	16.15607	726485	6942	9.555927	11737.14
60-64	169342	4116	24.30529	599732	8734	14.56282	14576.66
65-69	104998	3877	36.92904	524885	12286	23.40685	19383.5
70+	135268	12616	93.26414	904514	61192	67.65162	84358.72
Total	**5435734**	**53689**		**14576330**	**107064**		

CALCULATE TOTAL EXPECTED DEATHS

Country	Australia			Australia			
Year	1921			Standard 1981			Expected deaths in standard population based on ASDRs of 1921
	Population	Deaths	**ASDR**	Population	Deaths	**ASDR**	
0- 4	600206	11980	19.95991	1111945	2772	2.492918	22194.32
5- 9	595758	1168	1.960496	1249941	327	0.261942	2450.504
10-14	529040	795	1.502897	1295018	319	0.246041	1946.278
15-19	464217	985	2.120853	1259029	1075	0.853939	2670.215
20-24	454027	1385	3.050719	1247783	1260	1.009603	3806.636
25-29	462449	1734	3.7505	1184149	1089	0.919698	4441.15
30-34	449322	1951	4.341128	1192232	1076	0.90252	5175.632
35-39	387132	2168	5.599584	977233	1238	1.267231	5472.099
40-44	331518	2151	6.487855	822701	1674	2.034699	5337.565
45-49	280477	2376	8.469647	723139	2621	3.624794	6124.732
50-54	256066	2899	11.32285	757544	4459	5.885668	8577.556
55-59	215914	3488	16.15607	726485	6942	9.555927	11737.14
60-64	169342	4116	24.30529	599732	8734	14.56282	14576.66
65-69	104998	3877	36.92904	524885	12286	23.40685	19383.5
70+	135268	12616	93.26414	904514	61192	67.65162	84358.72
Total	**5435734**	**53689**		**14576330**	**107064**		**198252.7**

CALCULATE AGE STANDARDIZED DEATH RATE FOR 1921

Country	Australia			Australia			
Year	1921			Standard 1981			Expected deaths in standard population based on ASDRs of 1921
	Population	Deaths	**ASDR**	Population	Deaths	**ASDR**	
0- 4	600206	11980	19.95991	1111945	2772	2.492918	22194.32
5- 9	595758	1168	1.960496	1249941	327	0.261942	2450.504
10-14	529040	795	1.502897	1295018	319	0.246041	1946.278
15-19	464217	985	2.120853	1259029	1075	0.853939	2670.215
20-24	454027	1385	3.050719	1247783	1260	1.009603	3806.636
25-29	462449	1734	3.7505	1184149	1089	0.919698	4441.15
30-34	449322	1951	4.341128	1192232	1076	0.90252	5175.632
35-39	387132	2168	5.599584	977233	1238	1.267231	5472.099
40-44	331518	2151	6.487855	822701	1674	2.034699	5337.565
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60-64	169342	4116	24.30529	599732	8734	14.56282	14576.66
65-69	104998	3877	36.92904	524885	12286	23.40685	19383.5
70+	135268	12616	93.26414	904514	61192	67.65162	84358.72
Total	**5435734**	**53689**		**14576330**	**107064**		**198252.7**

Age standardized death rate for Australia in 1921 = 198252.7 / 1456330 * 1000 = 13.06

CALCULATE AGE STANDARDIZED DEATH RATE FOR 1921

Country	Australia			Australia			
Year	1921			Standard 1981			Expected deaths in standard population based on ASDRs of 1921
	Population	Deaths	**ASDR**	Population	Deaths	**ASDR**	
0- 4	600206	11980	19.95991	1111945	2772	2.492918	22194.32
5- 9	595758	1168	1.960496	1249941	327	0.261942	2450.504
10-14	529040	795	1.502897	1295018	319	0.246041	1946.278
15-19	464217	985	2.120853	1259029	1075	0.853939	2670.215
20-24	454027	1385	3.050719	1247783	1260	1.009603	3806.636
25-29	462449	1734	3.7505	1184149	1089	0.919698	4441.15
30-34	449322	1951	4.341128	1192232	1076	0.90252	5175.632
35-39	387132	2168	5.599584	977233	1238	1.267231	5472.099
40-44	331518	2151	6.487855	822701	1674	2.034699	5337.565
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60-64	169342	4116	24.30529	599732	8734	14.56282	14576.66
65-69	104998	3877	36.92904	524885	12286	23.40685	19383.5
70+	135268	12616	93.26414	904514	61192	67.65162	84358.72
Total	**5435734**	**53689**		**14576330**	**107064**		**198252.7**
Crude death rate		9.88			7.34		

Age standardized death rate for Australia in 1921 = 198252.7 / 1456330 * 1000 = **13.60**

- Crude death rate 1921 = 9.88
- Age standardized death rate for Australia in 1921 = 13.6

- Crude death rate 1981 (standard population) = 7.34

- If we were to account for the differing age structures, the mortality in 1921 is nearly twice (13.06 / 7.34 ≈ 2) the mortality in 1981

DIRECT STANDARDIZATION: SUMMARY

- Chose one of the comparison populations (1981) as standard
- Age Standardized Death Rate for Australia in 1921 can be directly compared against the CDR of 1981 population
- If we choose a third population as a standard, the we would
  - Calculate Age Standardized Death Rate for Australia in 1921
  - Calculate Age Standardized Death Rate for Australia in 1981
  - Compare the two Age Standardized Death Rates
- It is possible to standardize on more than one variable: eg. age-sex standardization
  - Requires knowledge of age-sex specific rates for all populations

CAUTIONS

- Choice of a standard can markedly alter comparisons between populations.

INTERPRETING STANDARDIZED RATES

- Standardized rates are artificial indices
- Should only be used for making comparisons
- Do not denote the actual events in the population
- Age is only one of the factors influencing mortality
  - Other factors not adjusted for by age standardization

- Always compare age-specific rates across populations
  - Is the pattern similar across various age-groups ?

DISADVANTAGES OF DIRECT STANDARDIZATION

- Requires knowledge the age/sex specific death rates for all populations being compared
- Such detailed data often not available

INDIRECT STANDARDIZATION

- Provides a measure called *standardized mortality ratio*
- Similar concept as direct standardization
- Requires knowledge of
  - Age/sex specific death rates of Standard Population
  - Total Deaths in Populations of interest
  - Age-structure of Populations of interest
- Does not Require Age-structure of deaths in Populations of Interest

AGE SPECIFIC DEATH RATES OF ONLY ONE POPULATION ARE KNOWN

Country	Australia		Australia	
Year	1921		1981	
	Population	Deaths	Population	Deaths
0- 4	600206	??	1111945	2772
5- 9	595758	??	1249941	327
10-14	529040	??	1295018	319
15-19	464217	??	1259029	1075
20-24	454027	??	1247783	1260
25-29	462449	??	1184149	1089
30-34	449322	??	1192232	1076
35-39	387132	??	977233	1238
40-44	331518	??	822701	1674
45-49	280477	??	723139	2621
50-54	256066	??	757544	4459
55-59	215914	??	726485	6942
60-64	169342	??	599732	8734
65-69	104998	??	524885	12286
70+	135268	??	904514	61192
Total	**5435734**	**53689**	**14576330**	**107064**
Crude death rate		9.88		7.34

POPULATION WITH KNOWN ASDR TAKEN AS STANDARD

			Standard Population		
Country	Australia		Australia		
Year	1921		1981		
	Population	Deaths	Population	Deaths	ASDR
0- 4	600206	??	1111945	2772	2.492918
5- 9	595758	??	1249941	327	0.261942
10-14	529040	??	1295018	319	0.246041
15-19	464217	??	1259029	1075	0.853939
20-24	454027	??	1247783	1260	1.009603
25-29	462449	??	1184149	1089	0.919698
30-34	449322	??	1192232	1076	0.90252
35-39	387132	??	977233	1238	1.267231
40-44	331518	??	822701	1674	2.034699
45-49	280477	??	723139	2621	3.624794
50-54	256066	??	757544	4459	5.885668
55-59	215914	??	726485	6942	9.555927
60-64	169342	??	599732	8734	14.56282
65-69	104998	??	524885	12286	23.40685
70+	135268	??	904514	61192	67.65162
Total	**5435734**	**53689**	**14576330**	**107064**	
Crude death rate		9.88		7.34	

EXPECTED DEATHS IN 1921 AS PER ASDR OF STANDARD POPULATION

				Standard Population		
Country	Australia			Australia		
Year	1921			1981		
	Population	Deaths	Expected Deaths	Population	Deaths	ASDR
0- 4	600206	??	1496	1111945	2772	2.492918
5- 9	595758	??	156	1249941	327	0.261942
10-14	529040	??	130	1295018	319	0.246041
15-19	464217	??	396	1259029	1075	0.853939
20-24	454027	??	458	1247783	1260	1.009603
25-29	462449	??	425	1184149	1089	0.919698
30-34	449322	??	406	1192232	1076	0.90252
35-39	387132	??	491	977233	1238	1.267231
40-44	331518	??	675	822701	1674	2.034699
45-49	280477	??	1017	723139	2621	3.624794
50-54	256066	??	1507	757544	4459	5.885668
55-59	215914	??	2063	726485	6942	9.555927
60-64	169342	??	2466	599732	8734	14.56282
65-69	104998	??	2458	524885	12286	23.40685
70+	135268	??	9151	904514	61192	67.65162
Total	**5435734**	**53689**	**23295**	**14576330**	**107064**	

Observed deaths in 1921	O	53689
Expected deaths in 1921	E	23295

				Standard Population		
Country	Australia			Australia		
Year	1921			1981		
	Population	Deaths	Expected Deaths	Population	Deaths	ASDR
0- 4	600206	??	1496	1111945	2772	2.492918
5- 9	595758	??	156	1249941	327	0.261942
10-14	529040	??	130	1295018	319	0.246041
15-19	464217	??	396	1259029	1075	0.853939
20-24	454027	??	458	1247783	1260	1.009603
25-29	462449	??	425	1184149	1089	0.919698
30-34	449322	??	406	1192232	1076	0.90252
35-39	387132	??	491	977233	1238	1.267231
40-44	331518	??	675	822701	1674	2.034699
45-49	280477	??	1017	723139	2621	3.624794
50-54	256066	??	1507	757544	4459	5.885668
55-59	215914	??	2063	726485	6942	9.555927
60-64	169342	??	2466	599732	8734	14.56282
65-69	104998	??	2458	524885	12286	23.40685
70+	135268	??	9151	904514	61192	67.65162
Total	**5435734**	**53689**	**23295**	**14576330**	**107064**	

Standardized Mortality Ratio	**SMR** = O / E	**2.30**

				Standard Population		
Country	Australia			Australia		
Year	1921			1981		
	Population	Deaths	Expected Deaths	Population	Deaths	ASDR
0- 4	600206	??	1496	1111945	2772	2.492918
5- 9	595758	??	156	1249941	327	0.261942
10-14	529040	??	130	1295018	319	0.246041
15-19	464217	??	396	1259029	1075	0.853939
20-24	454027	??	458	1247783	1260	1.009603
25-29	462449	??	425	1184149	1089	0.919698
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40-44	331518	??	675	822701	1674	2.034699
45-49	280477	??	1017	723139	2621	3.624794
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65-69	104998	??	2458	524885	12286	23.40685
70+	135268	??	9151	904514	61192	67.65162
Total	**5435734**	**53689**	**23295**	**14576330**	**107064**	

CDR for 1921	CDR_{1921}	9.88
CDR for 1981 - Standard Population	**CDRstd**	7.34
Observed deaths in 1921	O	53689
Expected deaths in 1921	E	23295
Standardized Mortality Ratio	**SMR** = O / E	**2.30**
Indirectly standardized death rate for 1921	**CDRstd** x **SMR**	**16.9**

INTERPRETATION OF INDIRECTLY STANDARDIZED

- Crude death rate 1981 (standard population) = 7.34
- Crude death rate 1921 = 9.88
- Indirectly standardized death rate 1921 = 16.9

  (for comparison, the directly standardized death rate for Australia in 1921 was 13.6)

- **Standardized Mortality Ratio = 2.30** (observed /expected deaths)
  - Observed deaths in 1921 were 2.3 times higher than what was expected based on 1981 death rates.
    - Thus mortality was higher in 1921
  - Direct standardization: 1.8 times higher mortality

DRAWBACKS OF INDIRECT STANDARDIZATION

- Indirect standardization does not keep age structure constant
  - SMR calculation dependent on age structure each compared population
  - The standard population has a separate age structure
  - If multiple populations are being compared, each SMR will be based on different population composition
- Direct standardization: all populations are brought to the same age structure as the standard population

MEASURES OF FERTILITY

MEASURES OF FERTILITY

PERIOD FERTILITY

- Events in current period studied in relation to durations of exposure of population during the same period
- Cross-Sectional / snapshot measures

COHORT FERTILITY

- Events studied in well defined cohorts as they move over time
- A long-run view of family building throughout a woman's reproductive years
- Two types:
  - Real cohorts
  - Synthetic / Hypothetical cohorts

PERIOD MEASURES OF FERTILITY

PERIOD MEASURES OF FERTILITY

- Total Births: in an year
- Rate of Natural Increase: (live births - deaths)/ mid-yr pop*1000
- Crude Birth Rate: live births/mid-yr pop * 1000
- Child-woman Ratio
- General Fertility Rate
- Age-specific Fertility Rate
- Age Specific Marital Fertility Rate

PERIOD MEASURES OF FERTILITY

- General Fertility Rate :

$$\frac{\text{live births in an year}}{\text{mid year population of women aged } 15-49} \times 1000$$

- General rate: All births attributed uniformly across all women aged 15-49 yr
- Sometimes age of women limited to 15-45 yr
- Requires: births as well as population estimates
- Hides age-specific variations

PERIOD MEASURES OF FERTILITY

- Crude Birth Rate
  - Not a rate but a ratio
  - Affected greatly by age-sex composition and other characteristics of population
  - Not suited for comparison among different populations

PERIOD MEASURES OF FERTILITY

- Age Specific Fertility Rate:

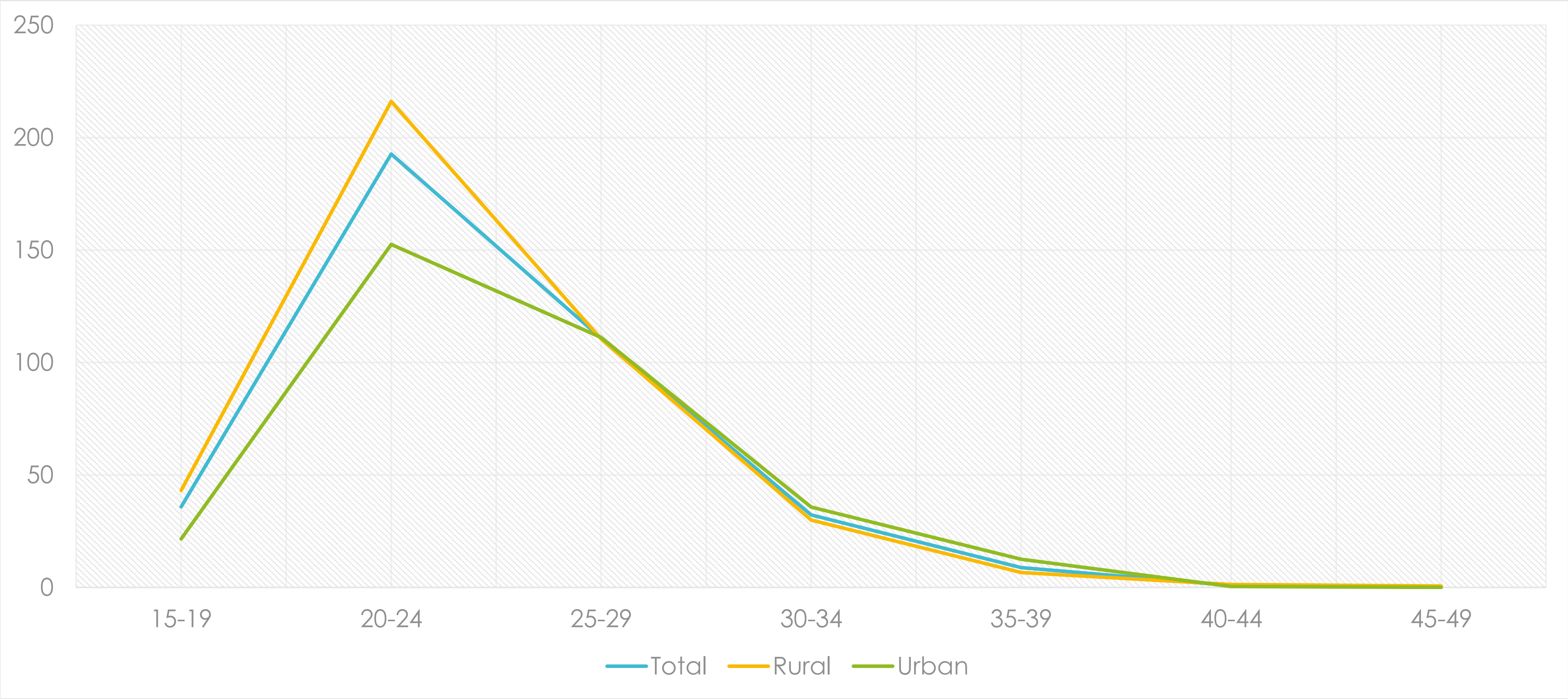
$$\frac{\text{live births in an year in women aged } x \text{ to } x+n}{\text{mid year population of women aged } x \text{ to } x+n} \times 1000$$

- Takes into account age related variations in fertility
- Most commonly reported as per five year age groups
  - Can also be reported as single year rate
- Small number of births taking place under 15 yr and > 49 yr added to the nearest age groups
- Requires: births as well as population estimates by age groups
- Enables comparison of fertility characteristics across populations:
  - Urban population: ASFRs will be more in older age groups

FERTILITY DATA IN SURVEYS: ISSUES AND CHALLENGES

- Information on complete birth histories of women ages 15-49 years
  - Accurateness and completeness information
  - Births of children who died very young may be under-reported
- NFHS-3
  - Birth histories for three years (2003-2005) preceding survey collected
    - For each married woman: births, age of woman at birth
  - ASFR for any specific age group
    - Numerator: number of births to women in that age group in
    - Denominator: Woman-years lived by women in that age group

AGE SPECIFIC FERTILITY RATES (KARNATAKA, 2011)



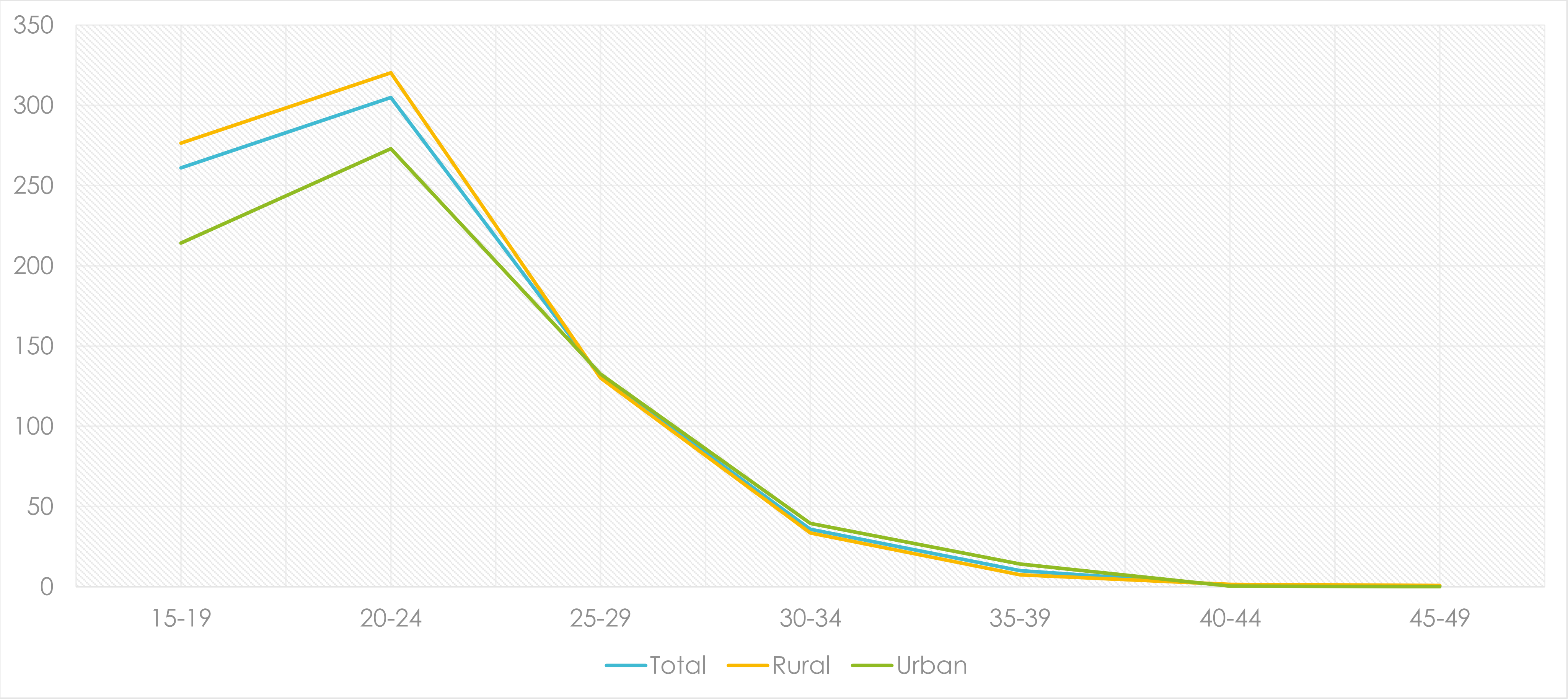
PERIOD MEASURES OF FERTILITY

- Age Specific Marital Fertility Rate :

$$\frac{\text{live births in an year in married women aged } x \text{ to } x+n}{\text{mid year population of married women aged } x \text{ to } x+n} \times 1000$$

- Takes into account age related variations in fertility by marriage status
- Reported as per five year age groups
- Small number of births taking place under 15 yr and > 49 yr added to the nearest age groups
- Requires: births as well as population estimates of married women by age groups
- Differences from the ASFRs can inform about the proportion of births occurring outside of marriage

AGE SPECIFIC MARITAL FERTILITY RATES (KARNATAKA, 2011)



PERIOD MEASURES OF FERTILITY

- Child-woman Ratio:

$$\frac{number\ of\ children\ aged\ 0-4}{number\ of\ women\ aged\ 15-49} \times 1000$$

- Enables fertility measurements when number of births is not available
- Helps compare fertility across geographically small areas
- Can be affected by mortality in women, migrations

MEASURES OF FERTILITY

- Child Woman Ratio: Alternate definition

$$\frac{number\ of\ children\ aged\ 10-14}{number\ of\ women\ aged\ 15-49} \times 1000$$

- Depicts the fertility trends in recent past adjusted for child mortality
- Problem in infant and child mortality differ substantially or significant under-reporting of child deaths

OTHER PERIOD MEASURES

- Crude marriage rate: marriages in year / mid-yr pop
- Crude divorce rate: divorces in year / mid-yr pop
- General Marriage rate: marriages in year / mid-yr 15+ pop
- General divorce rate: divorces in year / mid-yr 15+ pop
- Age specific marriage rate (females) : marriages of females in year in age group / mid-yr pop of females in age group
- Age specific divorce rate(females) : divorces of females in year in age group / mid-yr pop of females in age group

COHORT MEASURES OF FERTILITY

SYNTHETIC COHORT MEASURES OF FERTILITY

- Total Fertility Rate
- Total Marital Fertility Rate

- Gross Reproductive Rate
- Net Reproductive Rate

TOTAL FERTILITY RATE

- **Number of children a woman would bear during her reproductive years if she were to experience the current prevailing ASFRs**
- Average number of **births per woman per year**
- Sum of Age Specific Fertility Rates

$$TFR = 5 \sum_{i=15-19}^{45-49} f_i \,/1000$$

where, f_i = Age Specific Fertility Rates, for 5 yr age groups/ 1000 females group

- Multiplication by 5 required because ASFRs refer to births over five years (age group) while TFR refers to births per year
- Division by 1000 required since ASFRs are per 1000 women

TFR: A SOLVED EXAMPLE

Age Groups	Population	Births	ASFR (per 1000)
15-19	464217	3000	6.46
20-24	454027	5622	12.38
25-29	462449	2131	4.61
30-34	449322	6413	14.27
35-39	387132	4526	11.69
40-44	331518	3546	10.70
45-49	280477	2369	8.45
Total	**2829142**	**27607**	
		Sum of ASFRs	68.56
		TFR	= 5 * 68.56 / 1000 = 0.34

	NFHS-3, Urban India
Age Group	**ASFRs (per year)**
15-19	0.057
20-24	0.166
25-29	0.123
30-34	0.048
35-39	0.013
40-44	0.004
45-49	0.001
Sum of ASFRs	**0.412**
Age Group Width	5
TFR	= 0.412 * 5 = 2.06

Note: ASFRs are depicted as per woman per year. So in calculation of TFR, no need to divide by 1000

Age Specific Fertility Rates, India

Age	Urban	Rural	Total
15-19	0.057	0.105	0.09
20-24	0.166	0.231	0.209
25-29	0.123	0.146	0.139
30-34	0.048	0.069	0.062
35-39	0.013	0.031	0.025
40-44	0.004	0.009	0.007
45-49	0.001	0.004	0.003
TFR 15-44	2.06	2.96	2.66
TFR 15-49	2.06	2.98	2.68
CBR	18.8	25	23.1

Interpret the data ?

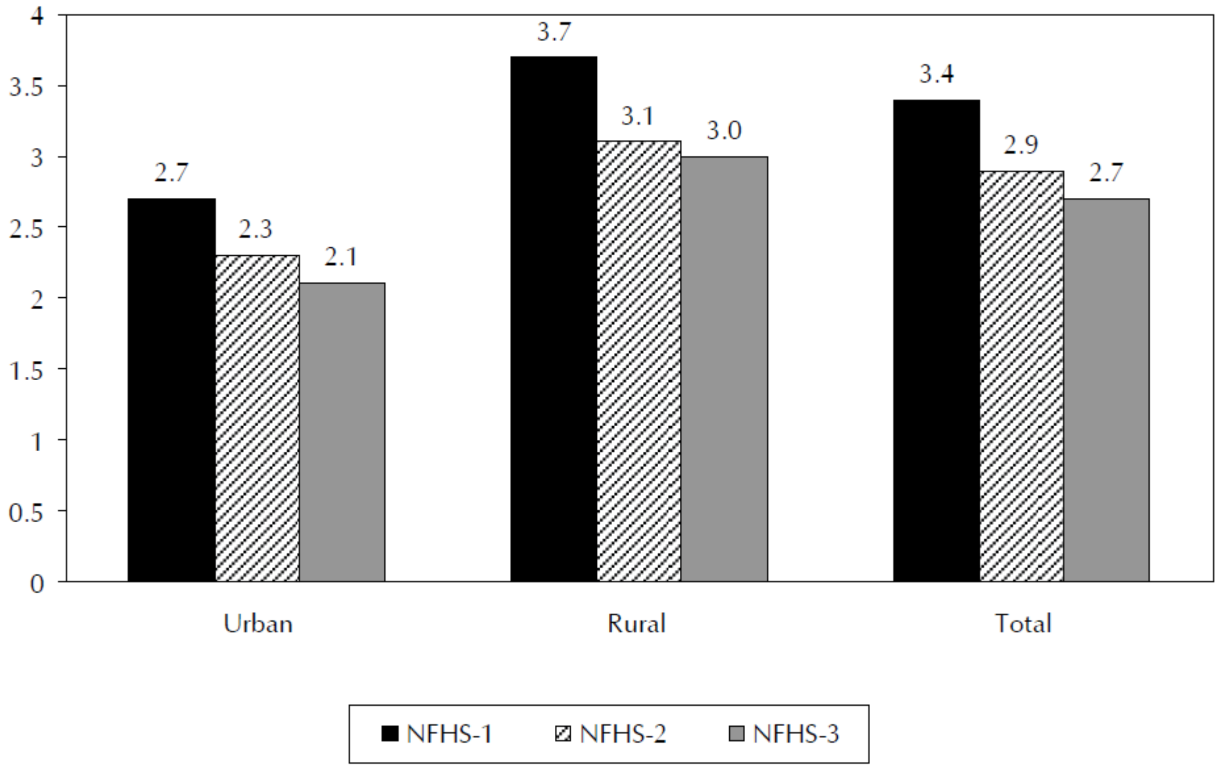
- The total fertility rate is almost one child higher in rural areas (3.0) than in urban areas (2.1).
- Age-specific fertility rates are lower at all ages in urban areas than in rural areas.
- Seventy percent of urban total fertility and 63 percent of rural total fertility are concentrated in the prime childbearing ages 20-29.
- There is also a moderate amount of early childbearing at age 15-19.
- Fertility at age 15-19 accounts for 14 percent of total fertility in urban areas and 18 percent in rural areas.
- Fertility at ages 35 and older accounts for only 4 percent of total fertility in urban areas and 7 percent in rural areas.

Source: NFHS-3

ASFR AND TFR IN INDIA TRENDS BASED ON NFHS DATA

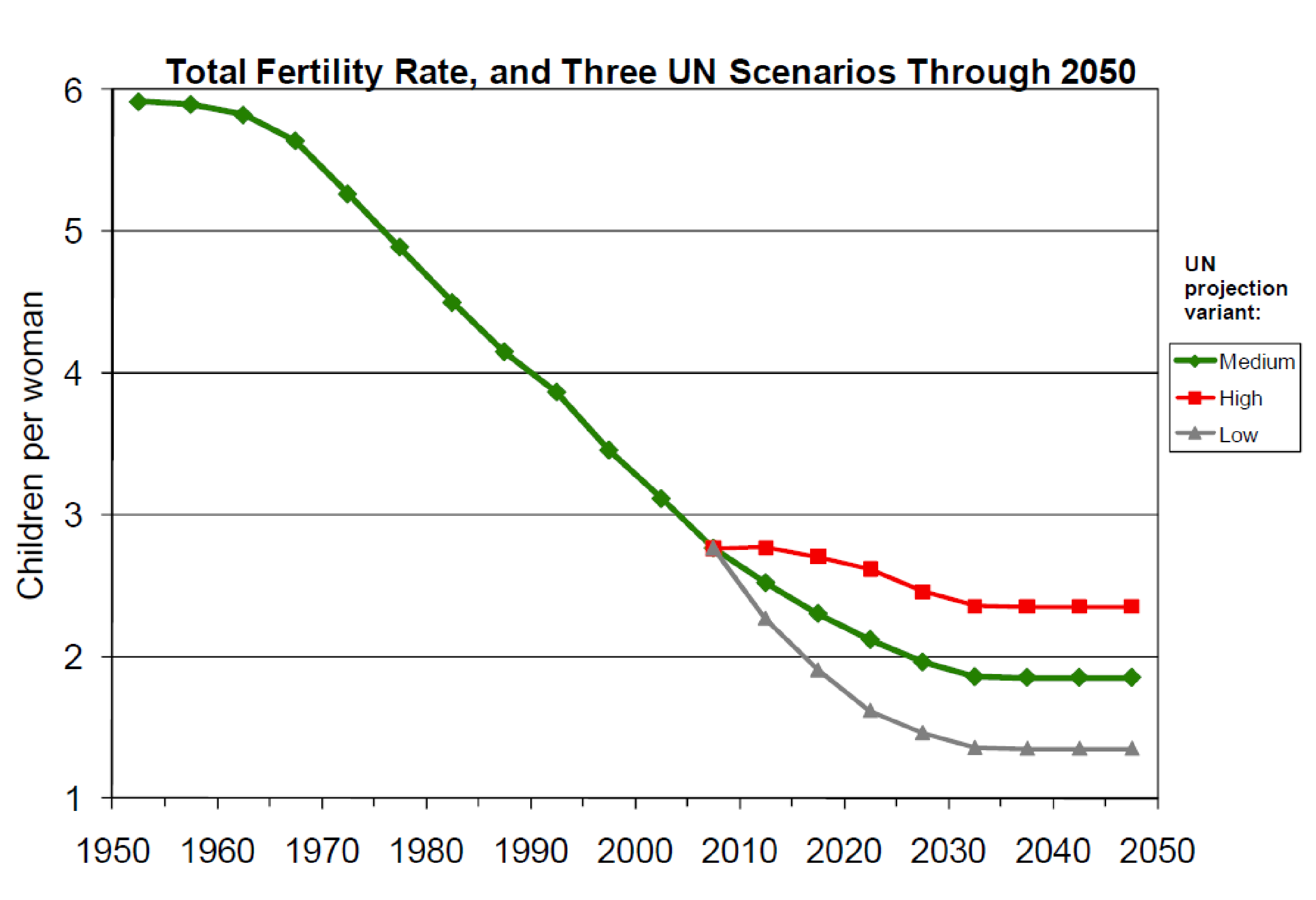
Age Group	NFHS 3	NFHS 2	NFHS 1
15-19	0.09	0.107	0.116
20-24	0.209	0.21	0.231
25-29	0.139	0.143	0.17
30-34	0.062	0.069	0.097
35-39	0.025	0.028	0.044
40-44	0.007	0.008	0.015
45-49	0.003	0.003	0.005
TFR 15-44	2.66	2.84	3.36
TFR 15-49	2.68	2.85	3.39

Figure 4.2 **Trends in Total Fertility Rates by Residence**



REPLACEMENT LEVEL TFR

- TFR of 2.1 often referred to as "*replacement level fertility*"
- An average of two children will replace each couple (mother and father)
  - Each couple replaced by a *couple*
  - Only if the children survive to reproductive age
  - Extra "0.1" added to 2
    - to account for mortality
    - Unbalanced sex-ratios
- If child mortality very high, the replacement level TFR would be higher
  - Was as high as 6 in pre-transition populations


Total Fertility Rate, and Three UN Scenarios Through 2050
Children per woman
6
5
4
3
2
1
1950
1960
1970
1980
1990
2000
2010
2020
2030
2040
2050
UN projection variant:
Medium
High
Low

TFR: DISADVANTAGES

- Does not measure complete fertility
  - Assumes that the current ASFRs will remain constant in future
  - Variations in ASFRs may occur due to changes in timing of births without changing total fertility
  - Sudden changes in fertility may occur: eg. Baby –boomers after WW2
- Ignore marriage status
  - Measures average number of children per woman
- Does not account for mortality
  - Children may die not survive till reproductive years
- Different sets of ASFRs may yield same TFR
  - Masks age-specific differences in fertility
- Does not by itself measure whether the fertility is above of below replacement: better indicators available

TOTAL MARITAL FERTILITY RATE

- **Number of children a married woman would bear during her reproductive years if she were to experience the current prevailing ASFRs**
- Average number of **births per married woman per year**
- Sum of Age Specific Marital Fertility Rates

$$TFR = 5 \sum_{i=15-19}^{45-49} f_i /1000$$

where, f_i = Age Specific Marital Fertility Rates, for 5 yr age groups/ 1000 females group

- Multiplication by 5 required because ASMFRs refer to births over five years (age group) while TMFR refers to births per year
- Division by 1000 required since ASMFRs are per 1000 women

GROSS REPRODUCTIVE RATE

- Average number of daughters borne to women if they follow current age-specific female fertility rates
- Sum of Age Specific Female Fertility Rates

$$GRR = 5 \sum_{i=15-19}^{45-49} {f^d}_i \,/1000$$

where, ${f^d}_i$= Age Specific Fertility Rates for daughter, for 5 yr age groups/ 1000 women

- Alternatively $GRR = TFR \, * proprotion\ of\ female\ births$
- Same disadvantages as TFR

NET REPRODUCTIVE RATE

- Average number of daughters borne to women if they follow current age-specific female fertility rates **and mortality rates**

$$NRR = 5 \sum_{i=15-19}^{45-49} f^d{}_i \times \left(\frac{{}_5L_x}{5 \times lo} \right) / 1000$$

Where

$f^d{}_i$= Age Specific Fertility Rates for daughter, for 5 yr age groups/ 1000 women

$\frac{{}_5L_x}{l_o} = \text{Probability of survival of daughter to the (mid point of) mother's age group}$

NRR

- Estimates the number of daughters who will live to replace their mothers in the future
- Allows for mortality between birth to the age of mother at the time of bearing the child
  - Mortality till daughter becomes the same age that her mother was when daughter was born
- Interpretation
  - NRR =1 : exact replacement
  - NRR >1: above replacement; growth ; future generation of potential mothers will be bigger than the one that produced them
  - NRR < 1: below replacement; decline ; future generation of potential mothers will be smaller than the one that produced them

NRR

- Assumes that mortality will change little over time
  - Assumption often correct in more developed countries
- If Mortality high
  - NRR << GRR
- If mortality low
  - NRR ≈ GRR

MEASURES OF FERTILITY: COMPARED

Indicator	Numerator	Denominator	Multiplier
GFR	Births in 1 yr	Females 15-49	1000
ASFR	Births to women in age group X	Women in Age group X	1000
TFR	[(Σ ASFRs)x group interval size] / 1000		
GMFR	Births in one year	Married females 15-49	1000
GIFR	Illegitimate Births in 1 year	Single, widowed, divorced, separated women 15-49 yrs	1000

INDICES OF MORTALITY

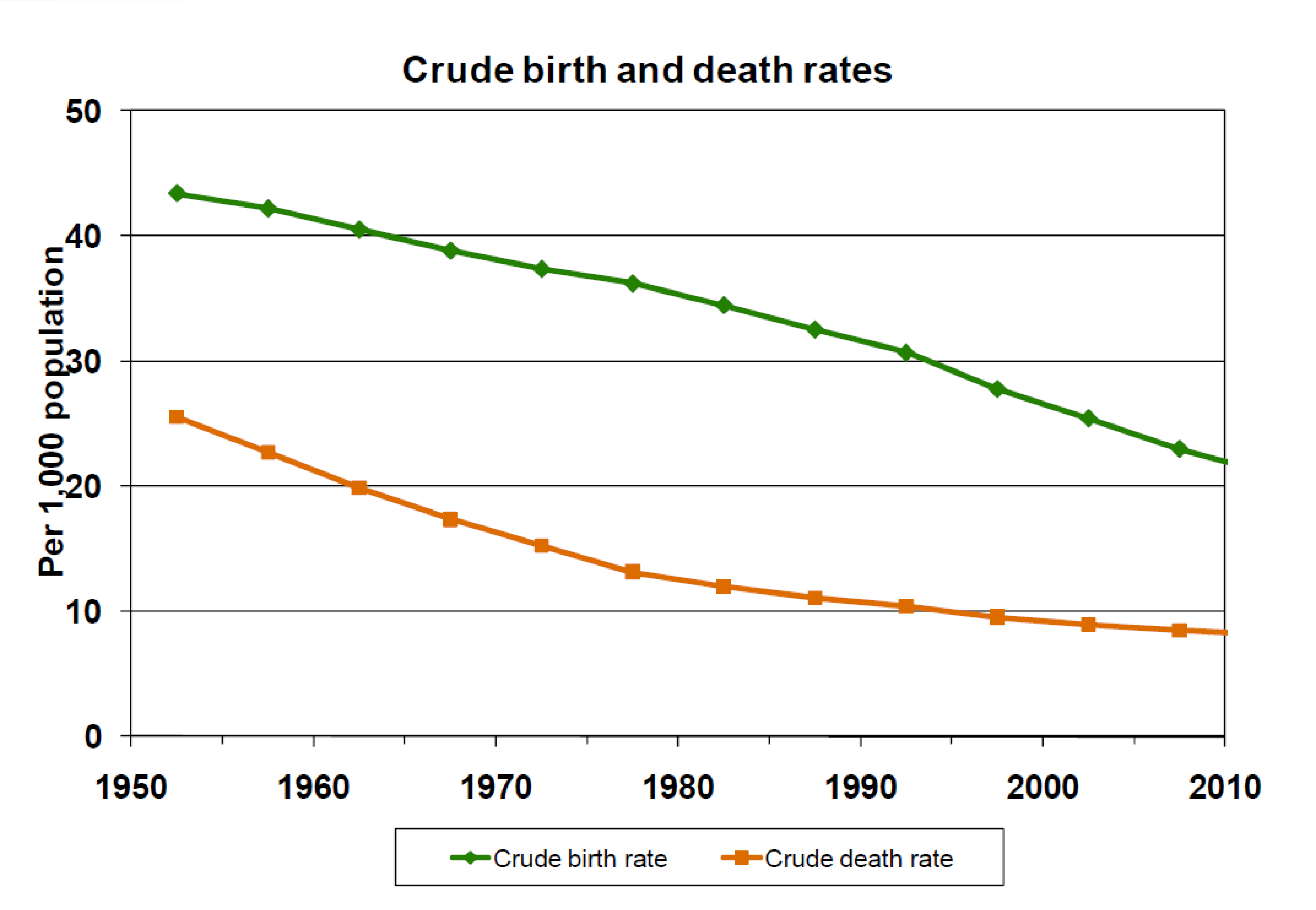
MORTALITY INDICATORS

- Number of deaths
- Crude death rate
- Age specific death rate
- Infant mortality rate
- Neonatal mortality Rate
- Post-neonatal mortality rate
- Perinatal mortality rate
- Stillbirth rate

INDICES OF MORTALITY

$$\text{Crude Death Rate (CDR)} = \frac{Deaths\ \ in\ a\ year}{Mid\ year\ population} \times 1000$$

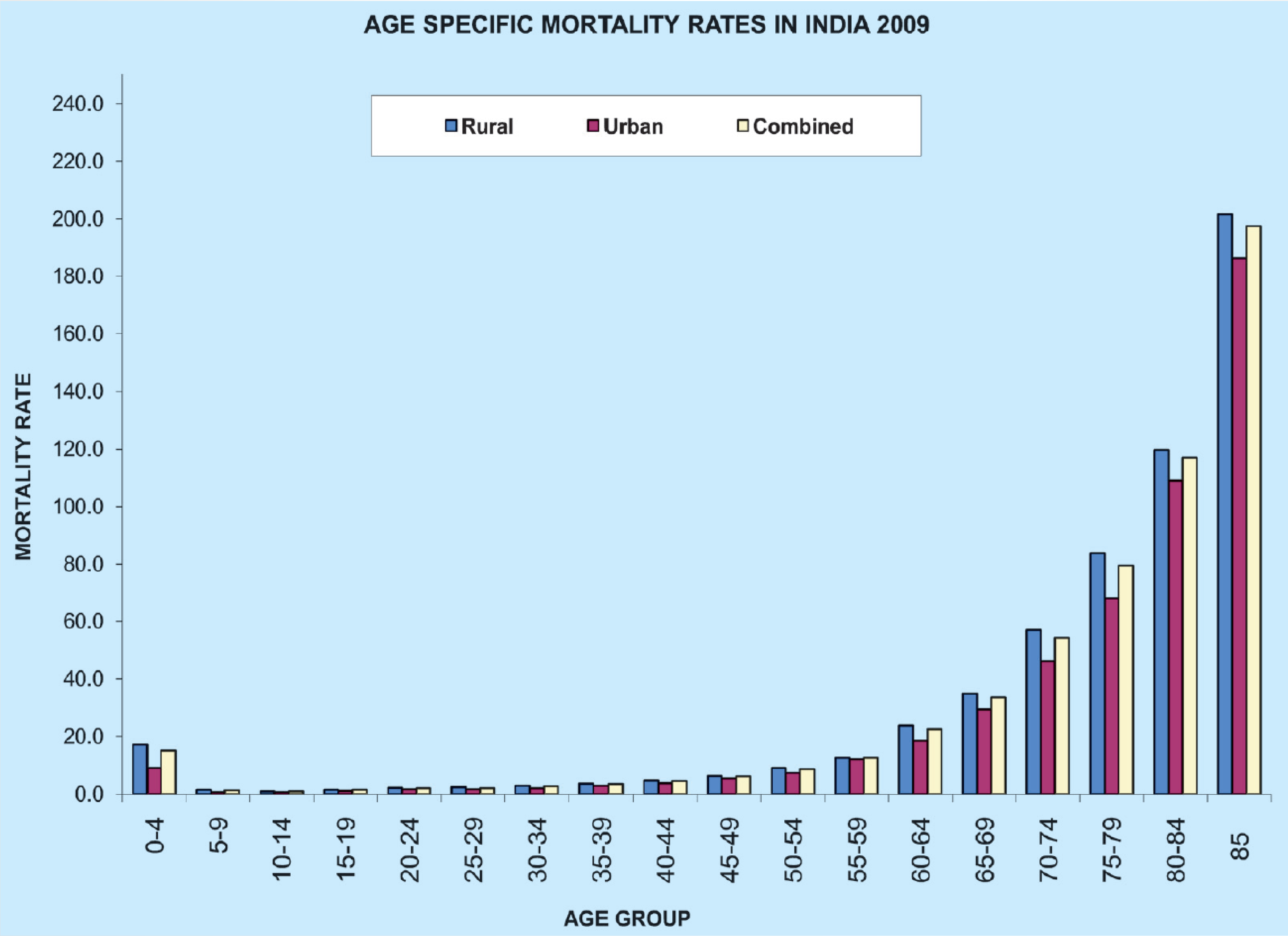
- The denominator is total population that has varying risks of death
- Does not take age structure into account
- Many developing countries have lower CDR than developed countries – most of population is young with low risk of death


Crude birth and death rates
50
40
30
20
10
0
Per 1,000 population
1950
1960
1970
1980
1990
2000
2010
Crude birth rate
Crude death rate

- Age Specific Death Rates

$$\frac{\text{deaths in an year in age group } x \text{ to } x+n}{\text{mid year population in age group } x \text{ to } x+n} \times 1000$$

- J shaped Graph



National Health Profile 2011

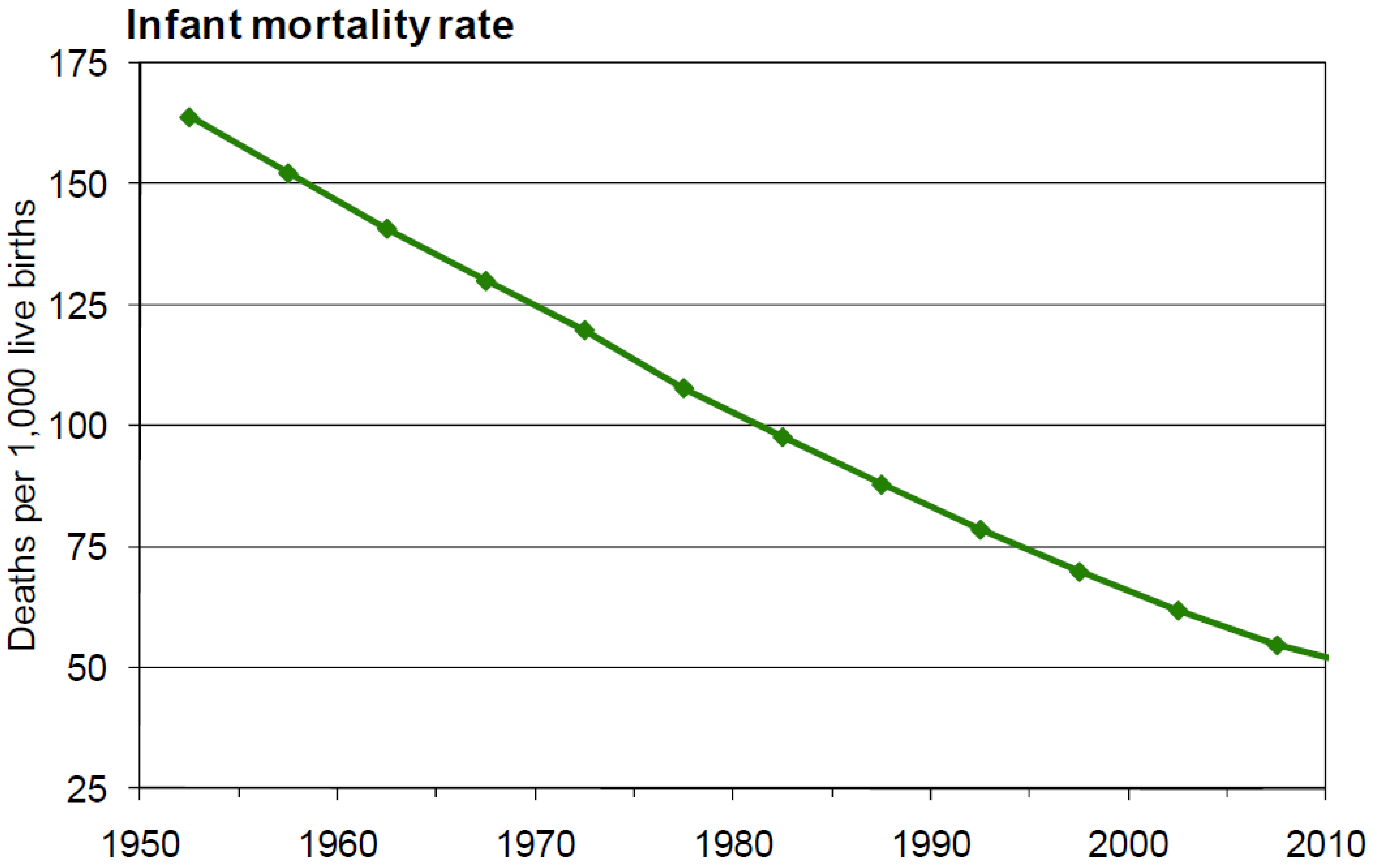
INDICES OF MORTALITY

- Infant Mortality Rate

$$\text{IMR} = \frac{\text{deaths under 1 year of age}}{\text{total live births in a calender year}} \times 1000$$

- Different from age-specific mortality rate
- Mid year population of 0-1 yr not an appropriate denominator
  - Majority of deaths in initial month: deaths not evenly distributed over the year
  - Not a good indicator of average size of population at risk for infant mortality

- Infant mortality rate
  - A ratio and not a rate
    - Migrations may affect the numerator but will not affect the denominator
    - Denominator includes births in calendar year while deaths may take place among children born in previous year
- Often presented as three-year moving averages
    - to smoothen year-to-year variations, and
    - to overcome effects of births occurring in calendar year while deaths including children born in previous year
- Considered sensitive indicator of the status of health of community



Infant mortality rate
Deaths per 1,000 live births
175
150
125
100
75
50
25
1950
1960
1970
1980
1990
2000
2010

INDICES OF MORTALITY

- Neonatal Mortality Rate

$$NMR = \frac{\text{deaths within first 28 } days \text{ of age}}{\text{total live births in a calender year}} \times 1000$$

  - Early- Neonatal Mortality Rate

$$NMR = \frac{\text{deaths within first 7 } days \text{ of age}}{\text{total live births in a calender year}} \times 1000$$

  - Late Neonatal Mortality Rate

$$NMR = \frac{\text{deaths within } 7 - 28 \text{ } days \text{ of age}}{\text{total live births in a calender year}} \times 1000$$

- Post-Neonatal Mortality Rate

$$PNMR = \frac{\text{deaths within 29 } days\ to \text{ 1 } year \text{ of age}}{\text{total live births in a calender year}} \times 1000$$

INDICES OF MORTALITY

- Stillbirth Rate

$$\frac{\text{deaths between 28 } weeks\ gestation}{\text{total live births \& still births in a calender year}} \times 1000$$

- Perinatal Mortality Rate

$$\frac{\text{deaths between 28 } weeks\ gestation \text{ \& 7 } days\ after\ birth}{\text{total live births \& still births in a calender year}} \times 1000$$

- Lower limit depends on age of gestation after which babies can survive outside the womb
  - Many developed countries use 20-22 weeks
    - Surogates: birth weight of 400-500 grams; Crown-heel length of 25 cms
- Upper limit for perinatal mortality : 28 days in many countries
- Statistics on abortions and perinatal deaths often not available or mis-reported (abortions as perinatal deaths) or under-reported in developing countries

MATERNAL MORTALITY RATE

$$\frac{\text{All maternal deaths}}{\text{total live births in the period}} \times 1000$$

- WHO, 1992, ICD-10: Maternal Death:
  - Death of woman while pregnant or within 42 days of termination of pregnancy irrespective of the duration and site of pregnancy.
  - Death can stem from any cause related to or aggravated by the pregnancy, or its management but not from accidental or incidental causes.
- Denominator: may be modified to include all pregnancies
  - Data on abortions often not available
  - Data on still-births often not available
- If total births not known, the *ESTIMATED* live births can be used
- A Ratio and not a rate

MATERNAL MORTALITY RATE

- The most widely used measure of
  - Maternal death
  - Overall health
  - Status of women in society
  - Gender inequalities
  - Health services and systems
- Difficulties in measurement
  - Comparative rarity
  - Reluctance to report abortion related deaths
  - Problems of recall
  - Lack of medical attribution
  - Poor routine registration systems

MATERNAL MORTALITY RATE

- Often subject to artificial changes
  - Under-reported deaths
  - Mis-reported deaths
  - Improvement in reporting systems
- A decreases may not represent real improvements in situation unless the change is sustained over long term
- Any changes in MMR must be investigated in light of ongoing interventions and activties

District population	1,600,000
Live Births	40,000
Number of Maternal Deaths	**MMR (per 100,000 births)**
5	125
7	175
9	225
11	275
13	325

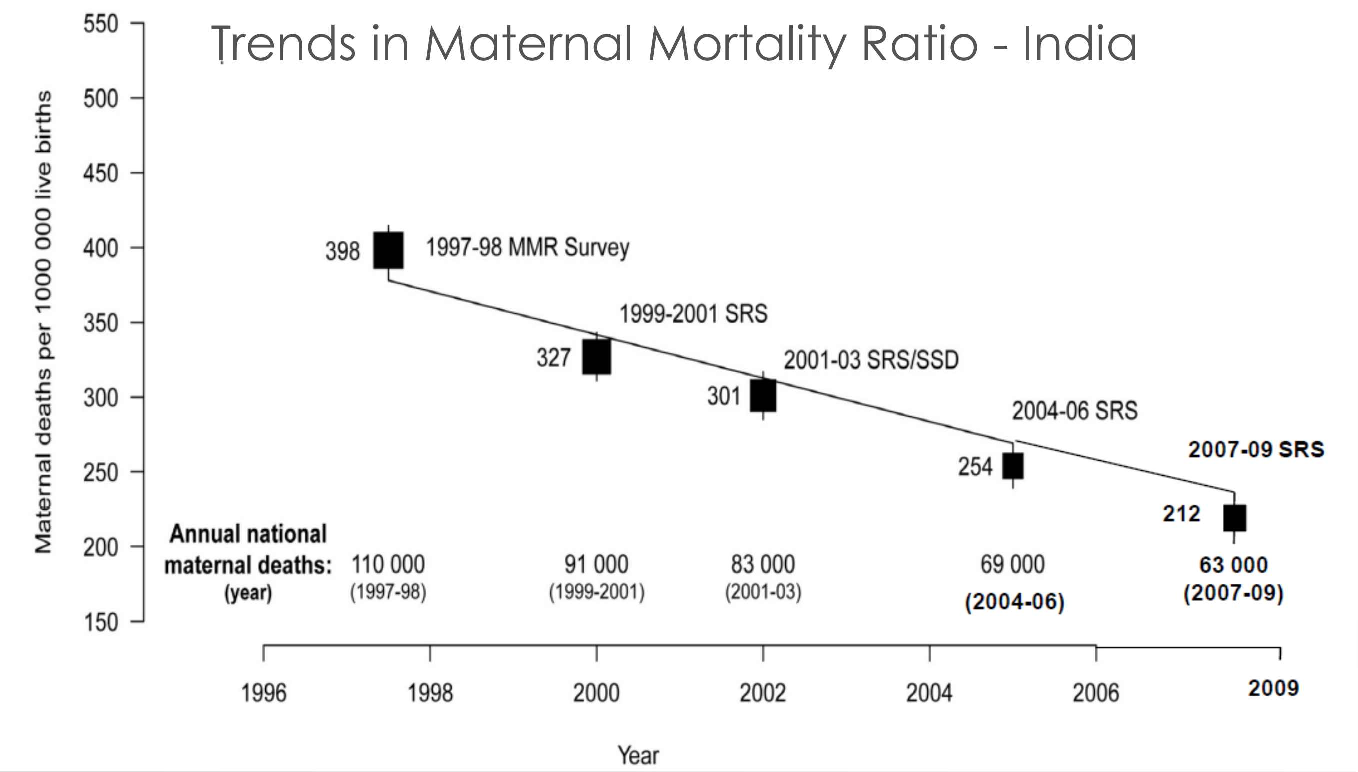
June, 2011

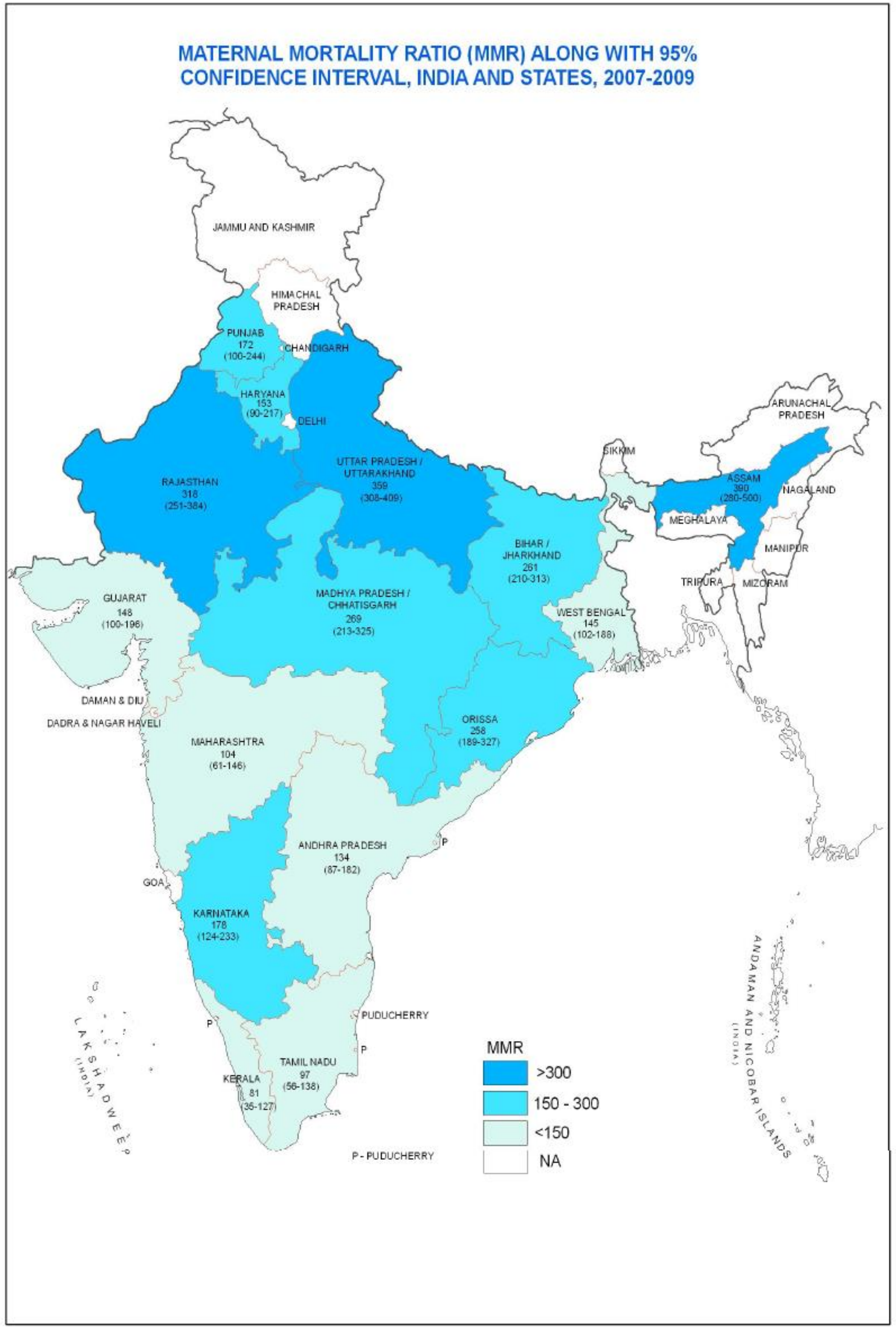
SPECIAL BULLETIN ON MATERNAL MORTALITY IN INDIA 2007-09

SAMPLE REGISTRATION SYSTEM

OFFICE OF REGISTRAR GENERAL, INDIA

VITAL STATISTICS DIVISION, WEST BLOCK 1, WING 1, 2ND FLOOR, R. K. PURAM, NEW DELHI-110 066





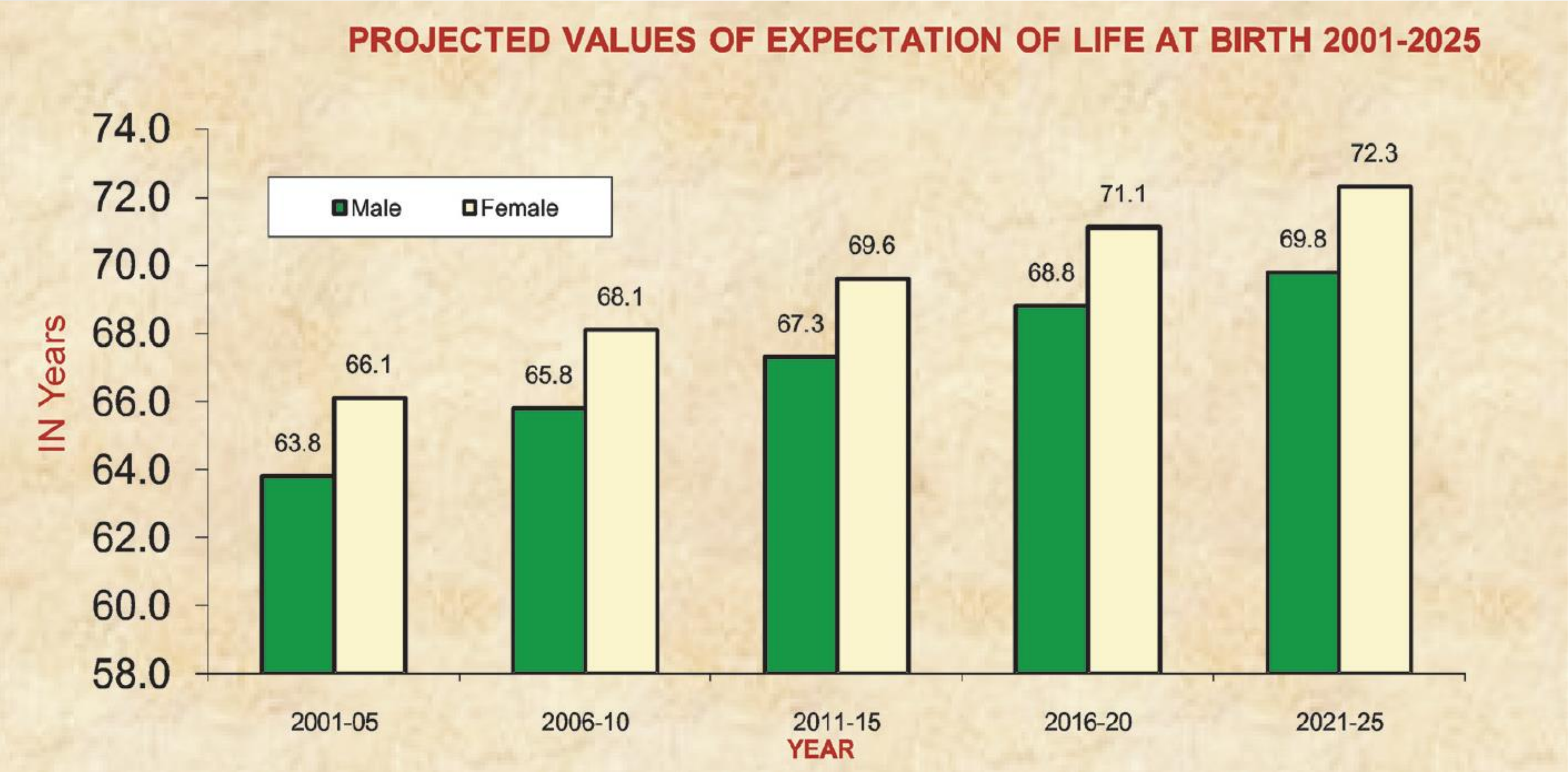
INDICES OF MORTALITY

- Cause specific death rates

$$\frac{\text{deaths in an year in } \textit{due to a specific cause}}{\text{total mid year population}} \times 1000$$

- Often calculated by sex and age

LIFE EXPECTATION AT BIRTH



LIFE TABLES

LIFE TABLES

- A statistical model that combines mortality rates at different age groups in a single setup
- Based on conditional probabilities
  - ${}_0P_n = {}_0P_{n-1} \times {}_{n-1}P_n$

  Where ${}_0P_n$ = total probability of surviving period n

  ${}_0P_{n-1}$ = total probability of survivng period n-1

  ${}_{n-1}P_n$ = probability of surviving nth period

LIFE TABLES

- Complete Life Table – life information is complete for each year of age
- Abriged life tables – Age groups used instead of single year
- Hypothetical Cohort table – based on age specific death rates in one year projecting them

SMOOTHENING OF AGE DISTRIBUTION

- Moving averages
  - For yearly data = Pn-1 + Pn +Pn+1 / 3
  - For 5 year age grouped data (Pn-1 + 2Pn + Pn+1) /4
- Use of polynomial and other curves

CREATING LIFE TABLES

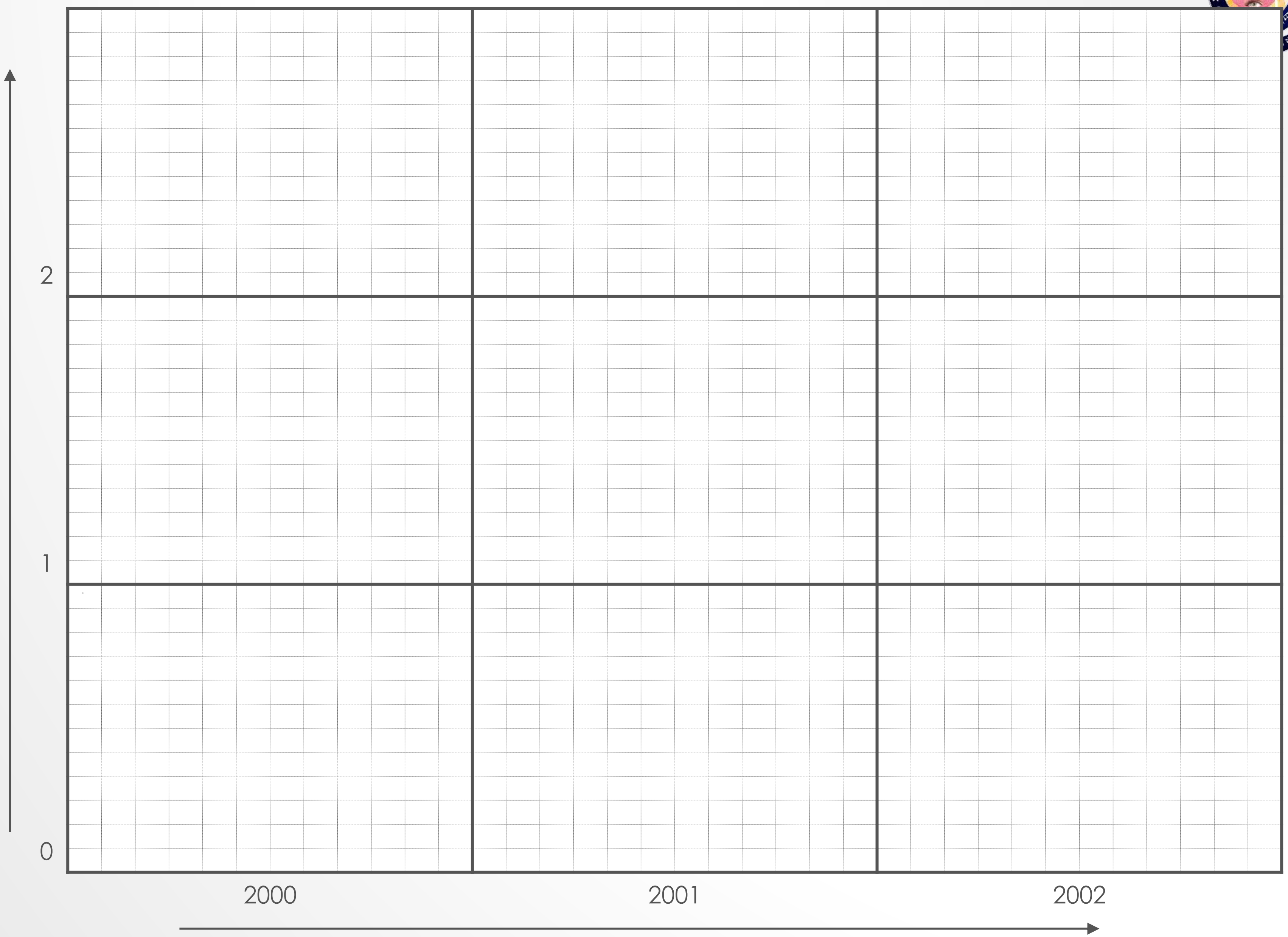
Subtitle

LEXIS DIAGRAMS

X Axis: Year

Y Axis: Duration/ Time

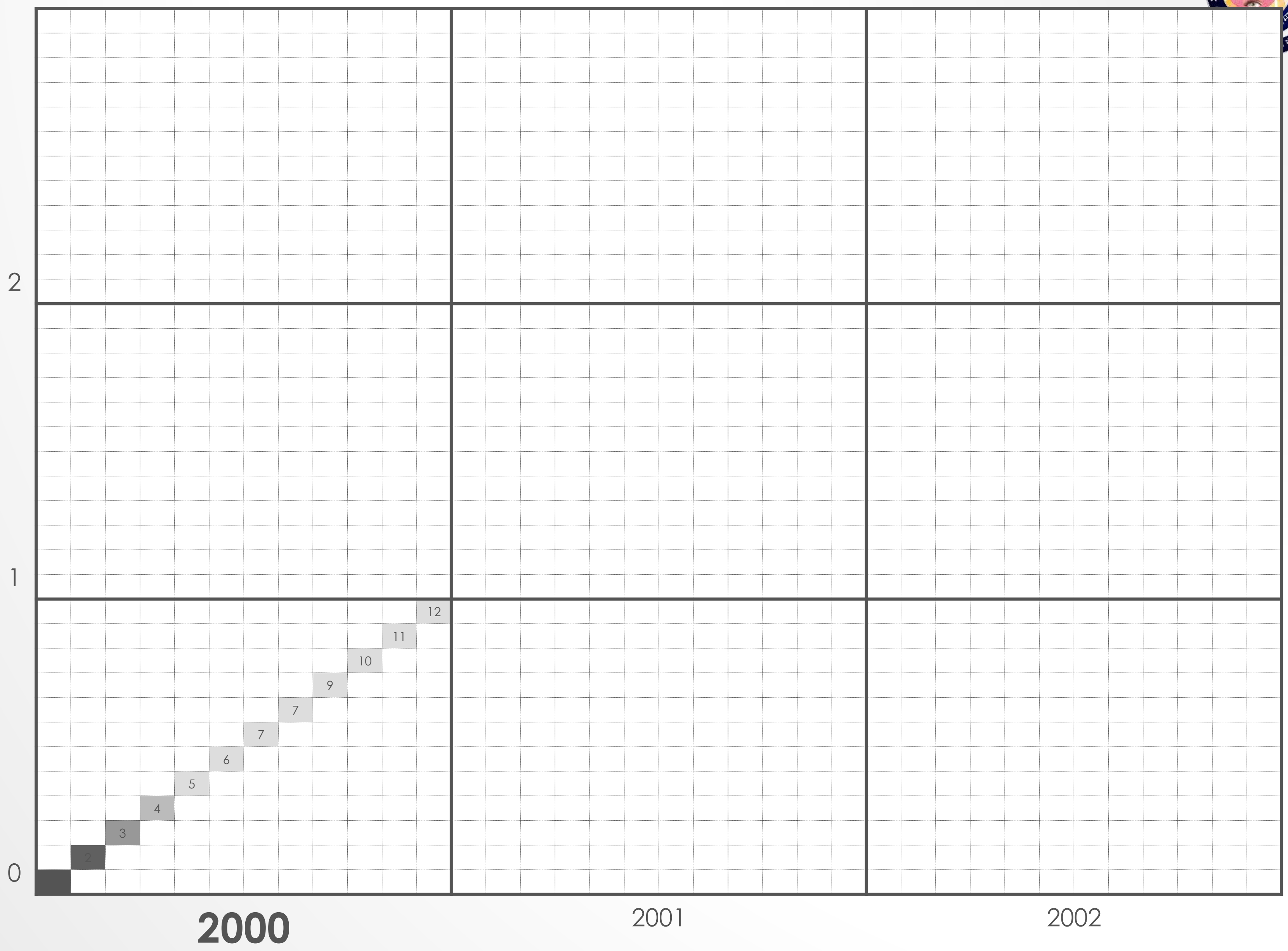
Each Small Box: 1 mth



LEXIS DIAGRAMS

Children born in year 2000

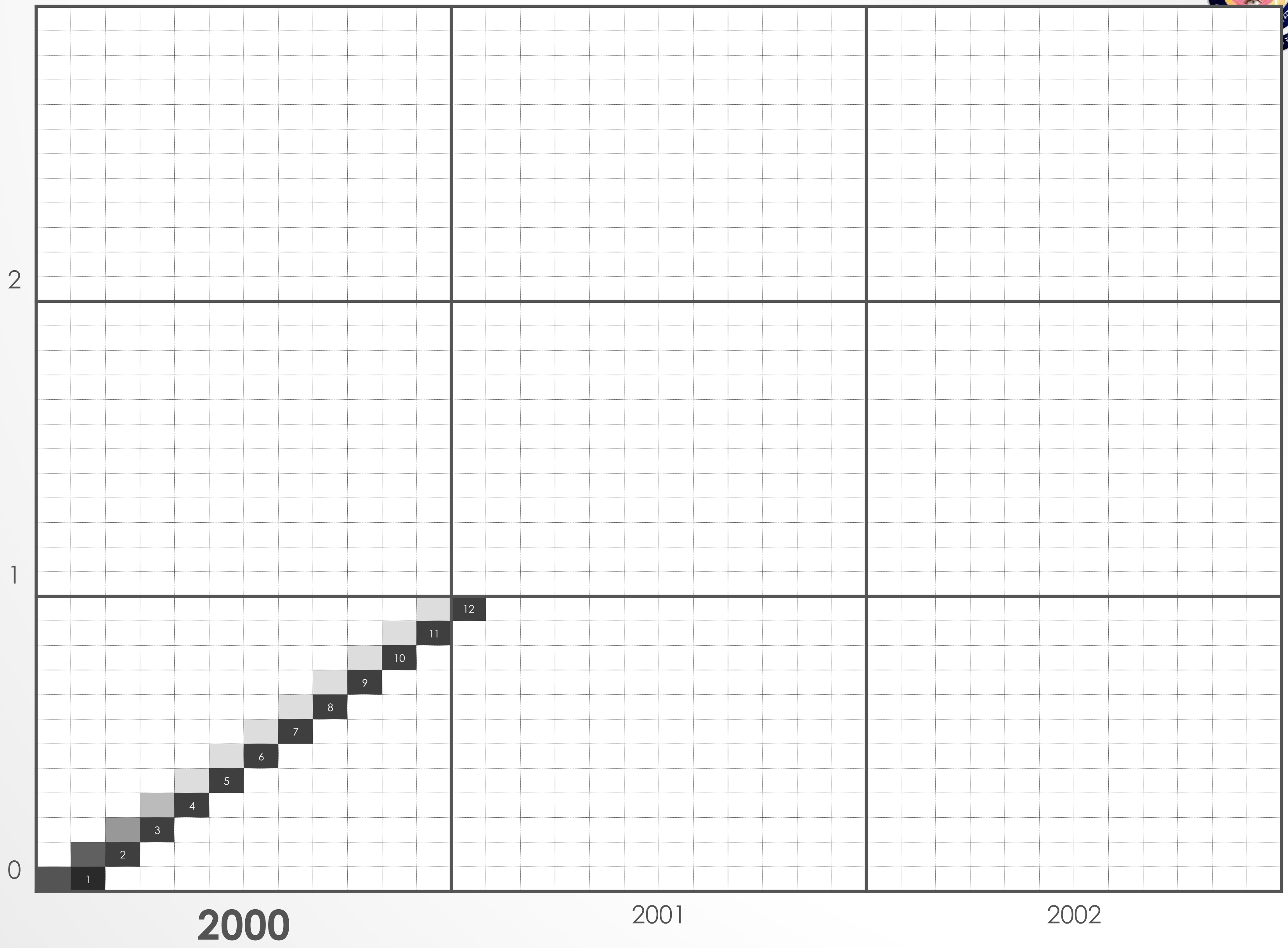
Born in first month of year 2000 → Track their progress over Time



LEXIS DIAGRAMS

Children born in year 2000

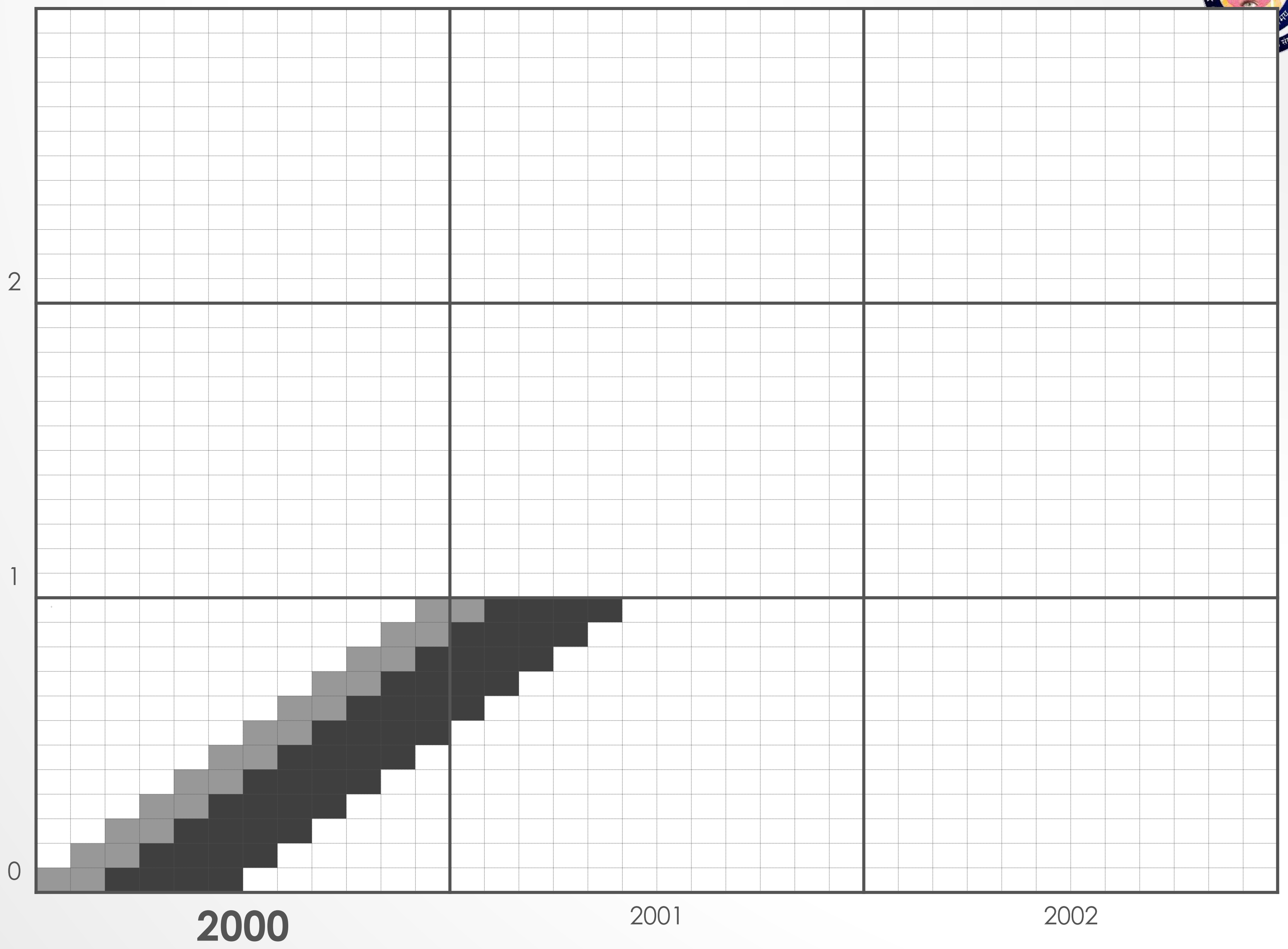
Born in Second month of year 2000 → Track their progress over Time



LEXIS DIAGRAMS

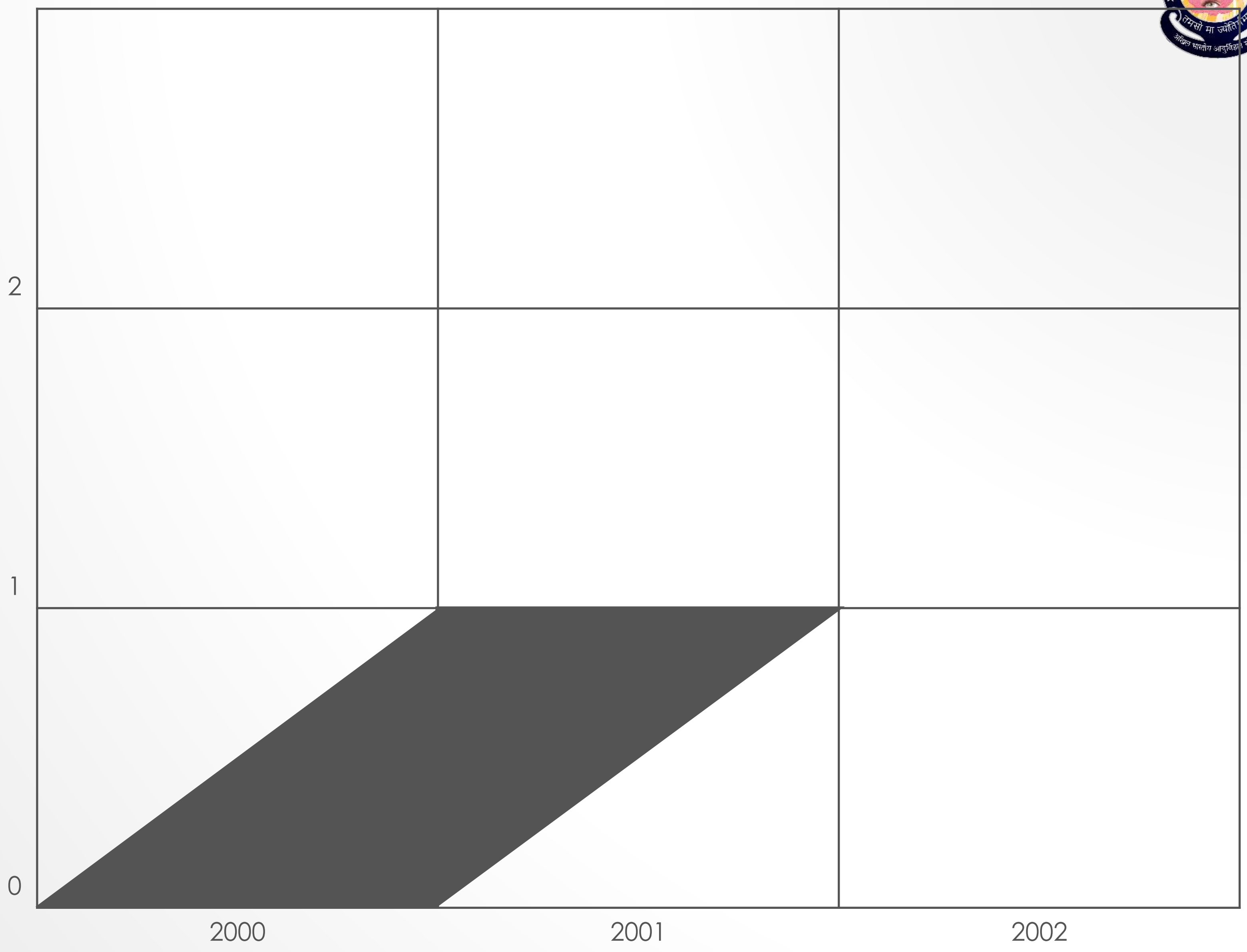
Children born in year 2000

Born in Third – sixth month of year 2000 → Track their progress over Time



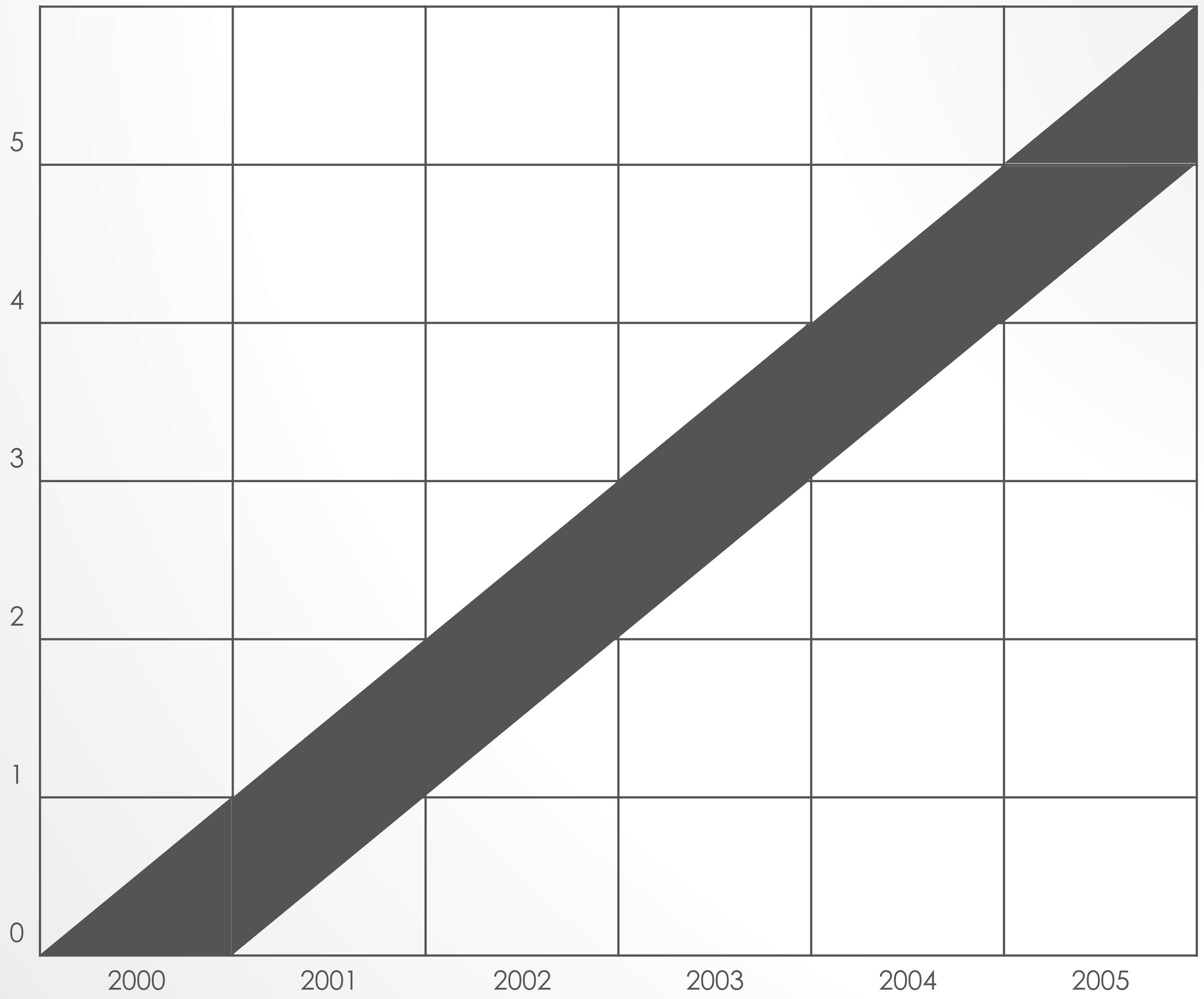
LEXIS DIAGRAMS

Shaded Area Represents
the Experience of Children born in year 2000 in first year of their life



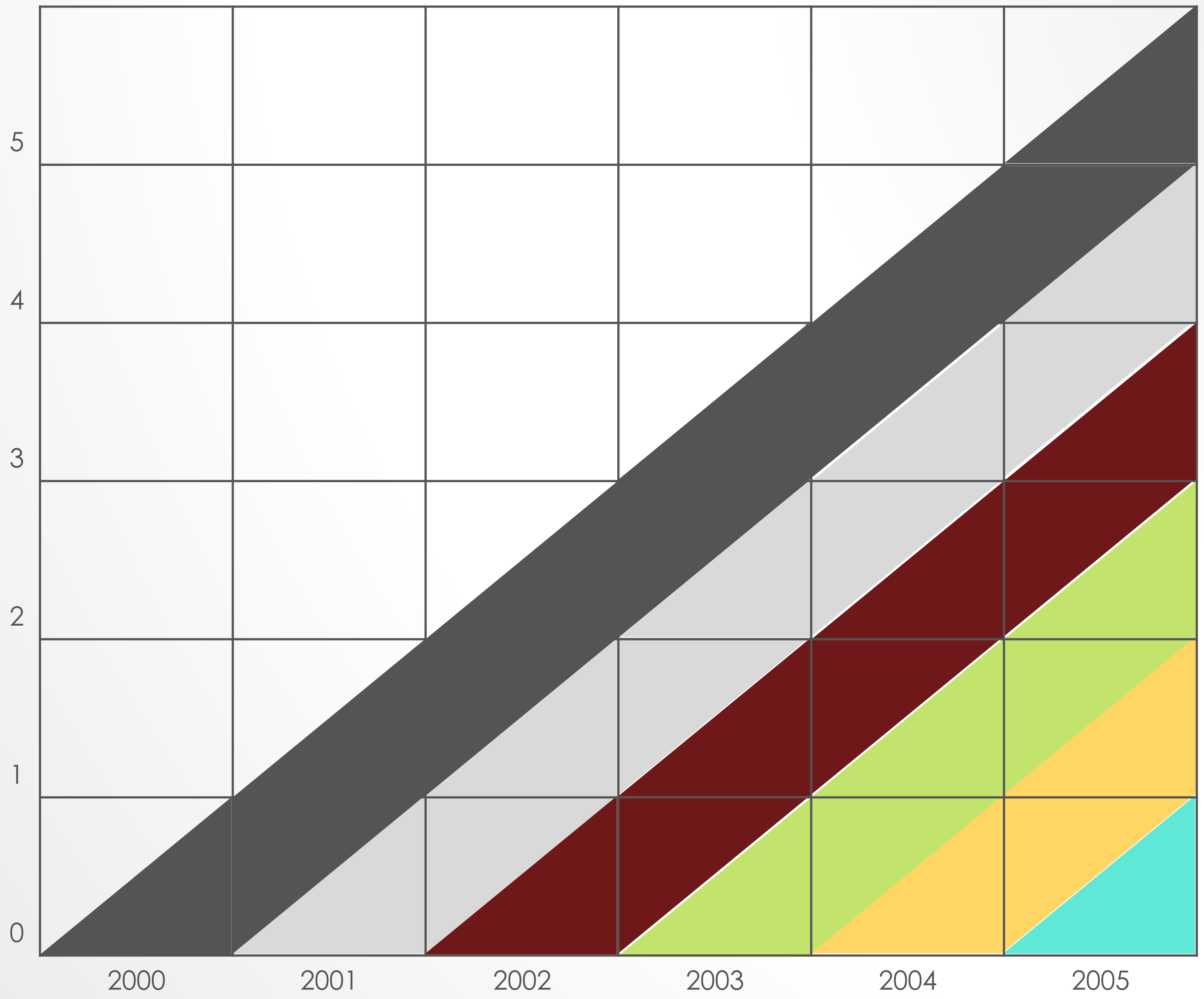
LEXIS DIAGRAMS

Shaded Area Represents the Experience of Children born in year 2000 in initial Five years of their life



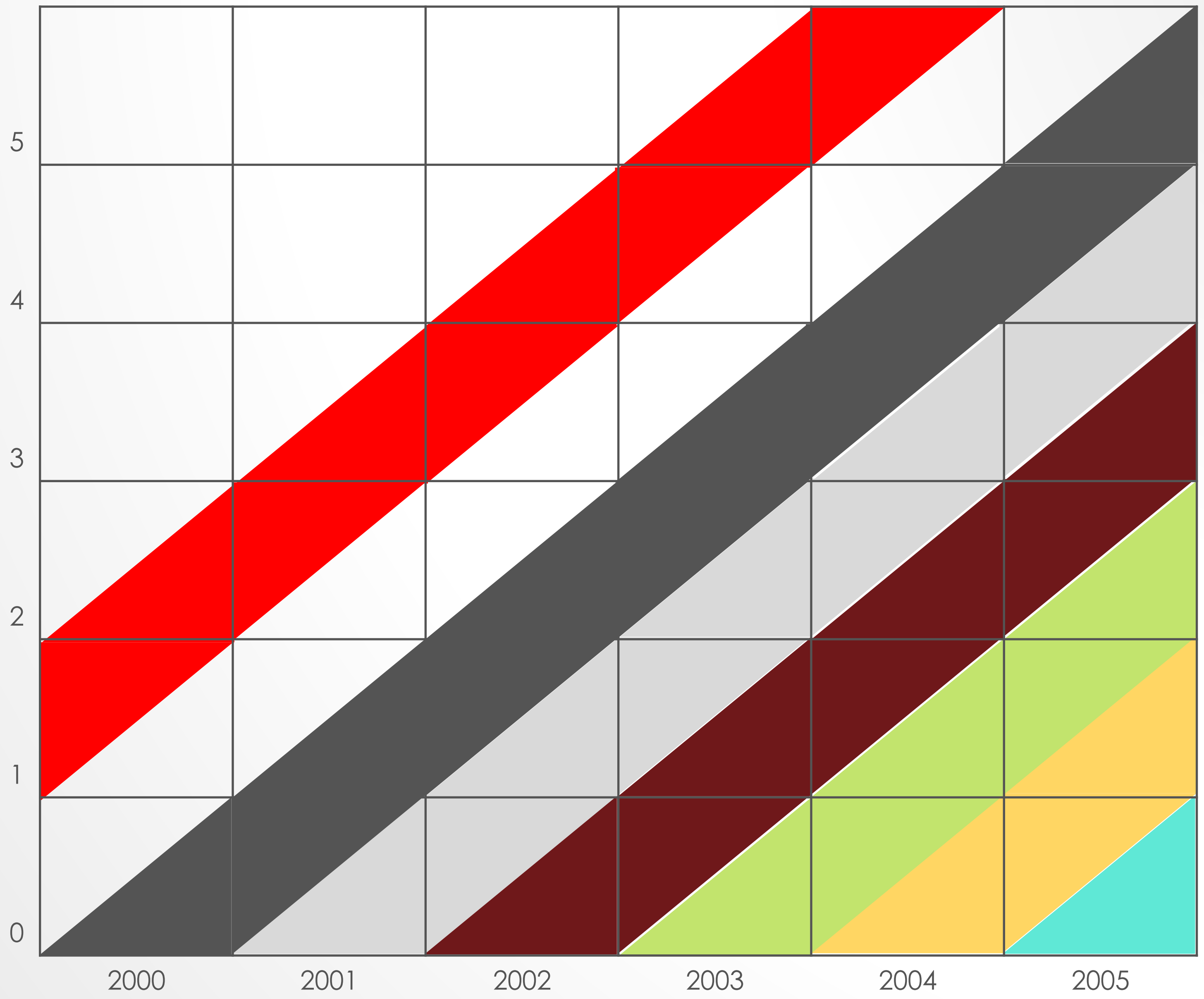
LEXIS DIAGRAMS

Each Shaded Area Represents
the Experience of individuals born in the corresponding base year



LEXIS DIAGRAMS

Which Area does the Bright Red band represent ?

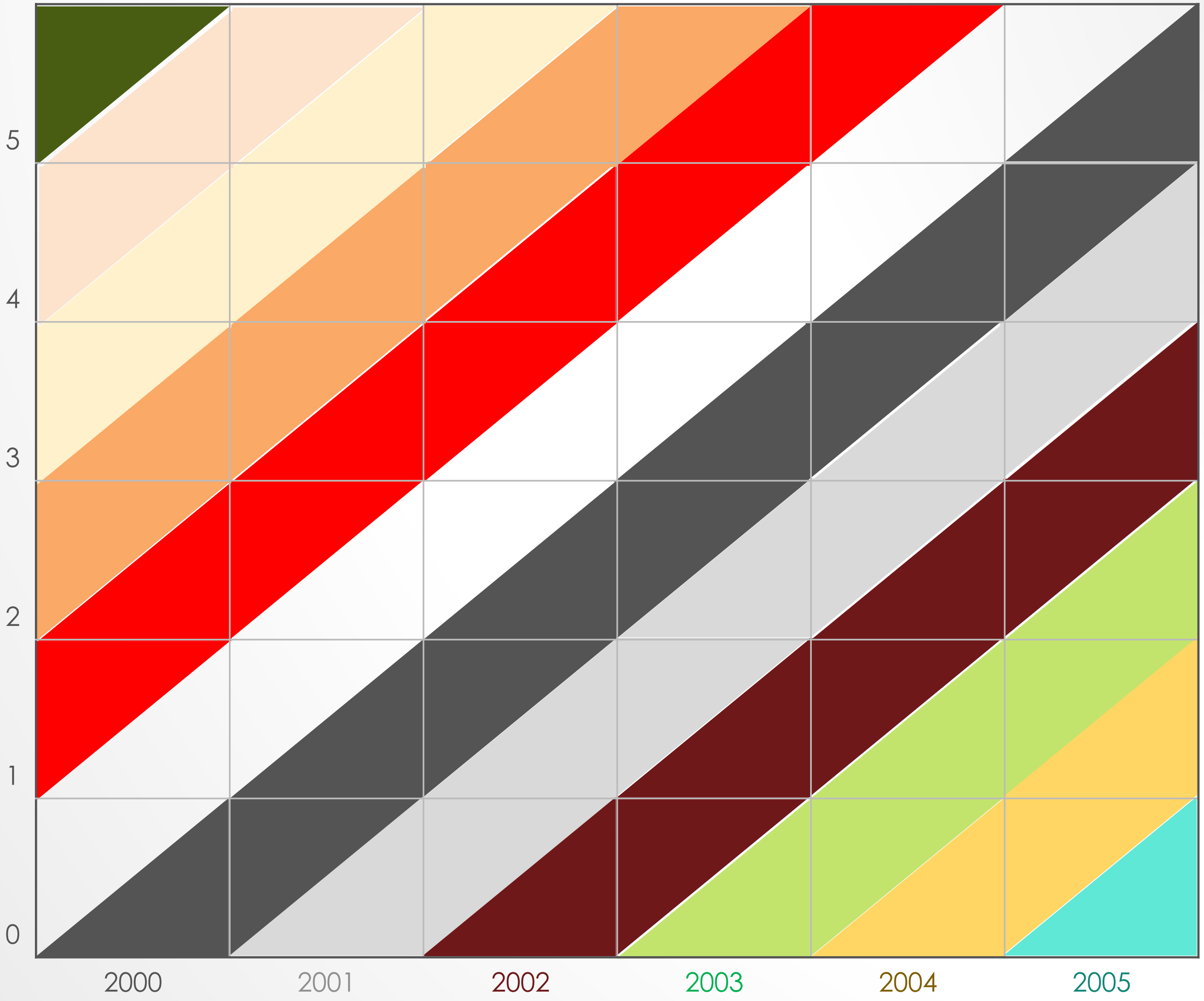


LEXIS DIAGRAMS

Each diagonal band represents a **Cohort** born in a particular year

Cohorts may have distinctive histories depending on the year of birth: upbringing, environmental influences, social change, access to care

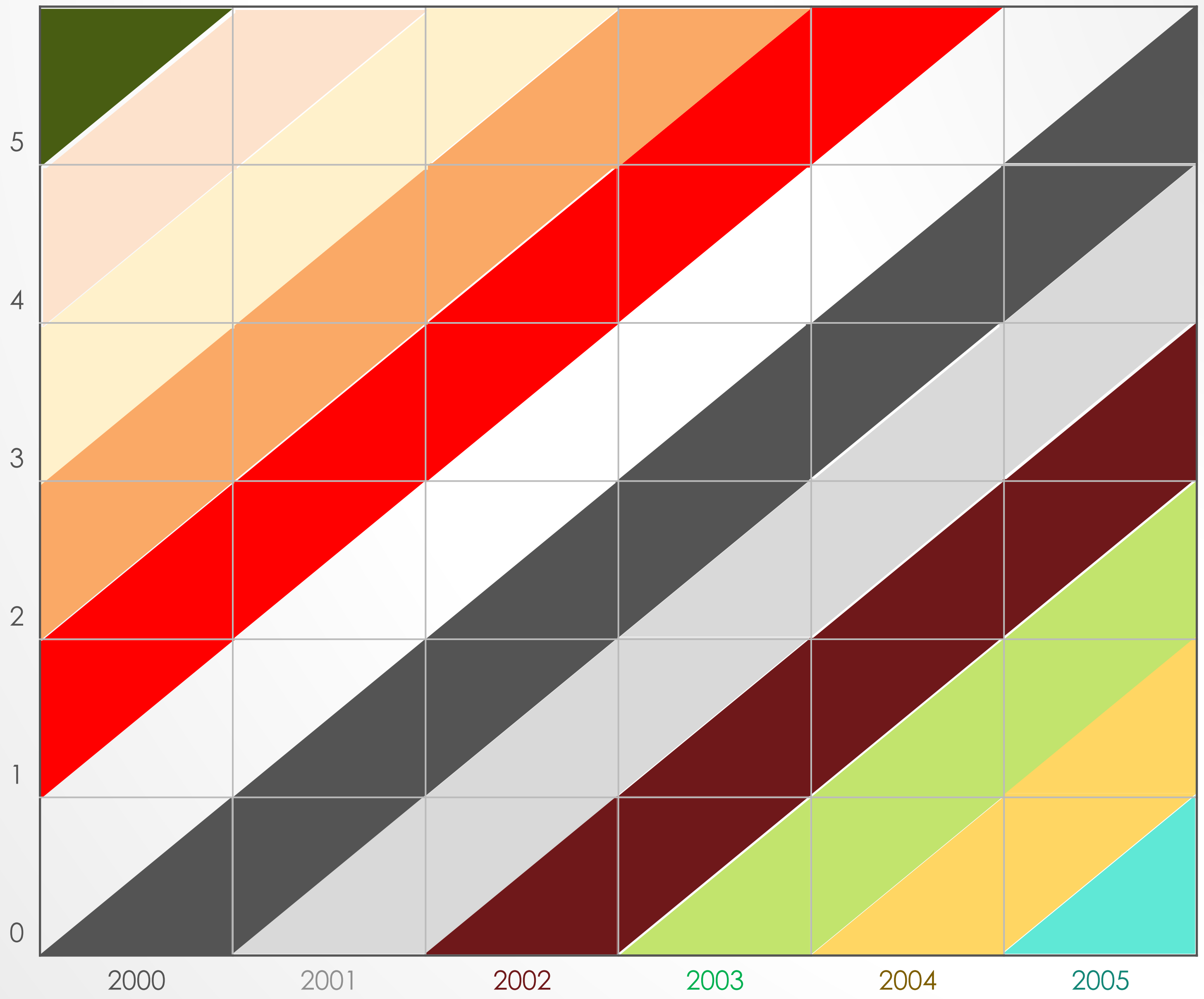
Eg India's 1982 birth cohort was not vaccinated with measles but majority of the 2012 cohort was vaccinated



LEXIS DIAGRAMS

Cohort Analysis

Traces the changing numbers and characteristics of cohorts over their lifetime



DISORDERED COHORT FLOW

- Sudden changes due to unusual characteristics of certain cohorts
  - Cohorts drastically differ from previous and subsequent cohorts
  - Eg
    - baby-boomers after the WW-2
    - War veterans
    - Immigrants

LEXIS DIAGRAMS

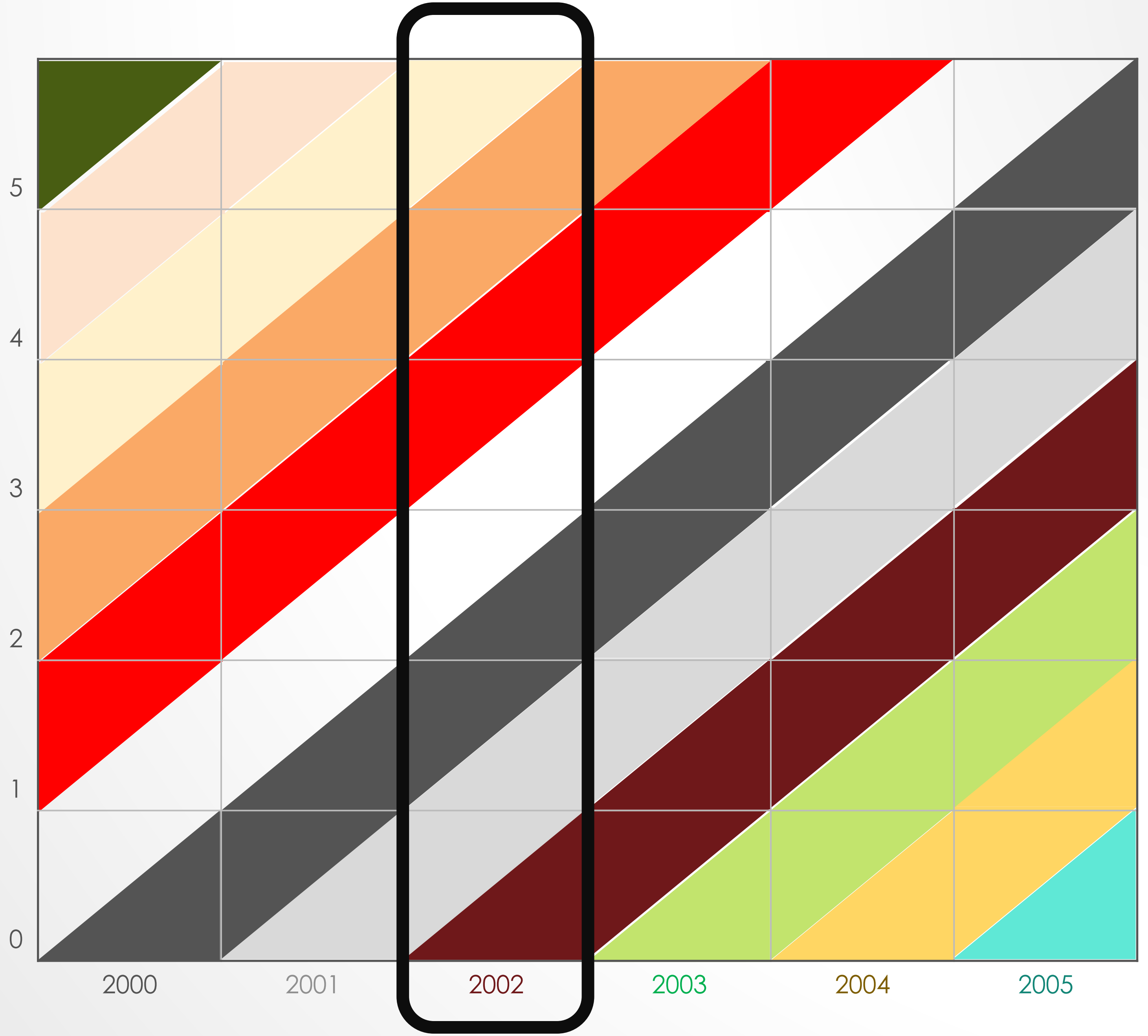
Period Analysis

A cross section of the population

Eg. Surveys, census

Mix of several cohorts

The information of all these cohorts is summarized to provide summary of the *current situation*. Eg Life expectancy, Total fertility rates



SYNTHETIC COHORT

- Cohorts based on period data
- Age specific Rates of current period are applied to a fictitious cohort or a synthetic cohort
  - The current period should NOT be an atypical period. Eg: epidemic, war, famine
  - Average data over several periods may be used to study the synthetic cohort to
- Behavior of fictitious cohort in subsequent periods assumed to be similar to the behavior in current period
  - A strong assumption that will definitely get violated: change is inevitable

LIFE TABLES

WHAT ARE LIFE TABLES

- First conceptualized by John Graunt
  - Number surviving at successive ages out of 100 ‘quick conceptions’ or live births
  - Using mortality data to obtain proportions surviving at each age
- Edmund Halley (1656-1742)
  - More rigorous mathematical approach to life tables
  - Life tables for the town of Breslau, Germany

TYPES OF LIFE TABLES

- Source of data
  - Period Life tables
  - Cohort life Tables

- Amount of detail that is available
  - Complete Life Table: each year of age is represented in table
  - Abridged Life table: age groups are represented in table

PERIOD LIFE TABLES

- Derived form age-specific mortality rates
  - Rates observed over one-year
  - Averages rates over multiple years

- Observed ASMRs applied to a *hypothetical / synthetic cohort*
  - Question of interest: How the hypothetical cohort would change over time if the observed ASMRs are applied to it ?

- Helps answer:
  - *In each nth year, how many from the cohort will die*
  - *At the end of nth years how many from the hypothetical would be alive*
  - *What would be the Life Expectancy at nth year, etc..*

UNDERLYING PRINCIPLES FOR PERIOD LIFE TABLES

- Stationary Population
  - Constant Size: Number of births = Number of Deaths
  - Constant Age Structure: for each stationary population
  - Closed to migration: no migration / net-migration is zero
- Assumptions often considered unrealistic
- Assumptions make calculations simple

CREATING ABRIDGED PERIOD LIFE TABLES: 1

- Observed Data on age-specific deaths
  - nNx = mid-period population in age interval x to $x + n$
  - nDx = deaths between ages x and $x + n$ during the period

- Calculate Age Specific Death Rates

$$ {}_nM_x = \frac{nDx}{nNx} \times k $$

Where: k = multiplier, per 1000 population / per 1 population

Age Group		nMx
Below 1	${}_1M_0$	38.7
1-4	${}_4M_1$	1.4
0-4	${}_5M_0$	8.6
5-9	${}_5M_5$	0.6
10-14	${}_5M_{10}$	0.5
15-19	${}_5M_{15}$	1.2
20-24	${}_5M_{20}$	1.6
25-29	${}_5M_{25}$	1.3
30-34	${}_5M_{30}$	2.9
35-39	${}_5M_{35}$	3.4
40-44	${}_5M_{40}$	3.6
45-49	${}_5M_{45}$	7.1
50-54	${}_5M_{50}$	8.1
55-59	${}_5M_{55}$	13.6
60-64	${}_5M_{60}$	18.8
65-69	${}_5M_{65}$	31.8
70-74	${}_5M_{70}$	44.8
75-79	${}_5M_{75}$	83
80-84	${}_5M_{80}$	108.1
85+	${}_\infty M_{85}$	169

CREATING ABRIDGED PERIOD LIFE TABLES: 2

- **Probability of Death** between ages x and $x + n$

$$ {}_nq_x = \frac{2n \times nMx}{2 + n \times nMx} $$

Where: nMx = ASDR per 1 population
nMx needs to be divided by 1000 if it was earlier calculated as per 1000 population

(derivation of formula of $_nq_x$)

- Everyone in last age group will have to die → $_\infty q_{85}$ will be 1
- Probability of death during 1 to 4.99 years = $_4q_1$
- Probability of death during 5 to 9.99 years = $_5q_5$

Age Group		$_nM_x$	n	x	$_nq_x$
< 1	$_1q_0$	38.7	1	0	0.03796537
1-4	$_4q_1$	1.4	4	1	0.005584364
5-9	$_5q_5$	0.6	5	5	0.002995507
10-14	$_5q_{10}$	0.5	5	10	0.002496879
15-19	$_5q_{15}$	1.2	5	15	0.005982054
20-24	$_5q_{20}$	1.6	5	20	0.007968127
25-29	$_5q_{25}$	1.3	5	25	0.006478943
30-34	$_5q_{30}$	2.9	5	30	0.014395632
35-39	$_5q_{35}$	3.4	5	35	0.016856718
40-44	$_5q_{40}$	3.6	5	40	0.017839445
45-49	$_5q_{45}$	7.1	5	45	0.034880865
50-54	$_5q_{50}$	8.1	5	50	0.039696153
55-59	$_5q_{55}$	13.6	5	55	0.065764023
60-64	$_5q_{60}$	18.8	5	60	0.089780325
65-69	$_5q_{65}$	31.8	5	65	0.147290412
70-74	$_5q_{70}$	44.8	5	70	0.201438849
75-79	$_5q_{75}$	83	5	75	0.3436853
80-84	$_5q_{80}$	108.1	5	80	0.42550679
85+	$_\infty q_{85}$	169	Infinity	85	1

ALETRNATIVE FORMULA FOR

$$ {}_nq_x = \frac{n \times {}_nM_x}{1 + (n - {}_na_x) \times {}_nM_x} $$

Where, ${}_na_x$ = Average years of survival within the age group

These have been provided by Coale, Ansley J, Demeny P. Regional Model Life Tables and Stable Populations, Princeton University Press, 1966.

And Chang CJ. Life Tables and Mortality Analysis. Geneva: World Health Organization; 1980.

CREATING ABRIDGED PERIOD LIFE TABLES: 3

- **Probability of Survival from** age x to $x + n$

$$_{n}p_{x} = 1 - {}_{n}q_{x}$$

- Everyone in last age group will have to die

  → $_{\infty}p_{85}$ will be 0

- Probability of surviving from 1 to 4.99 years = $_{4}p_{1}$
- Probability of surviving from 5 to 9.99 years = $_{5}p_{5}$

Age Group	$_{n}M_{x}$	$_{n}q_{x}$	$_{n}p_{x}$
< 1	38.7	0.03796537	0.96203463
1-4	1.4	0.005584364	0.994415636
5-9	0.6	0.002995507	0.997004493
10-14	0.5	0.002496879	0.997503121
15-19	1.2	0.005982054	0.994017946
20-24	1.6	0.007968127	0.992031873
25-29	1.3	0.006478943	0.993521057
30-34	2.9	0.014395632	0.985604368
35-39	3.4	0.016856718	0.983143282
40-44	3.6	0.017839445	0.982160555
45-49	7.1	0.034880865	0.965119135
50-54	8.1	0.039696153	0.960303847
55-59	13.6	0.065764023	0.934235977
60-64	18.8	0.089780325	0.910219675
65-69	31.8	0.147290412	0.852709588
70-74	44.8	0.201438849	0.798561151
75-79	83	0.3436853	0.6563147
80-84	108.1	0.42550679	0.57449321
85+	169	1	0

CREATING ABRIDGED PERIOD LIFE TABLES: 4

- **Number Surviving at exact ages x**

$$l_{x+n} = l_x \times {}_np_x$$

(small l, rounded off to whole numbers)

l_0 = Number surviving at age 0 = Initial population

l_1 = Number surviving at age 1 = Number surviving at age 0 **x** probability of surviving first year (0-0.99 yr)

$$l_1 = l_0 \times {}_1p_0$$

l_5 = Number surviving at age 5 = Number surviving at age 1 **x** probability of surviving 1-4 years (1-4.99 yr)

$$l_4 = l_1 \times {}_4p_1$$

Age Group	${}_nM_x$	${}_nq_x$	${}_np_x$
< 1	38.7	0.03796537	0.96203463
1-4	1.4	0.005584364	0.994415636
5-9	0.6	0.002995507	0.997004493
10-14	0.5	0.002496879	0.997503121
15-19	1.2	0.005982054	0.994017946
20-24	1.6	0.007968127	0.992031873
25-29	1.3	0.006478943	0.993521057
30-34	2.9	0.014395632	0.985604368
35-39	3.4	0.016856718	0.983143282
40-44	3.6	0.017839445	0.982160555
45-49	7.1	0.034880865	0.965119135
50-54	8.1	0.039696153	0.960303847
55-59	13.6	0.065764023	0.934235977
60-64	18.8	0.089780325	0.910219675
65-69	31.8	0.147290412	0.852709588
70-74	44.8	0.201438849	0.798561151
75-79	83	0.3436853	0.6563147
80-84	108.1	0.42550679	0.57449321
85+	169	1	0

CREATING ABRIDGED PERIOD LIFE TABLES: 4

- **Number Surviving at exact ages x**

$$l_{x+n} = l_x \times {}_np_x$$

We assume an initial population for our synthetic cohort : ***radix, lets say 100,000***

Then we keep on calculating l_x for each successive age group

REMEMBER: for calculations , ${}_np_x$ refers to

*probability of survival of **previous** period*

Can be interpreted as number entering any age group

Age Group	${}_nM_x$	${}_nq_x$	${}_np_x$	l_x
< 1	38.7	0.03796537	0.96203463	100000
1-4	1.4	0.005584364	0.994415636	96203
5-9	0.6	0.002995507	0.997004493	95666
10-14	0.5	0.002496879	0.997503121	95380
15-19	1.2	0.005982054	0.994017946	95142
20-24	1.6	0.007968127	0.992031873	94572
25-29	1.3	0.006478943	0.993521057	93819
30-34	2.9	0.014395632	0.985604368	93211
35-39	3.4	0.016856718	0.983143282	91869
40-44	3.6	0.017839445	0.982160555	90321
45-49	7.1	0.034880865	0.965119135	88709
50-54	8.1	0.039696153	0.960303847	85615
55-59	13.6	0.065764023	0.934235977	82216
60-64	18.8	0.089780325	0.910219675	76810
65-69	31.8	0.147290412	0.852709588	69914
70-74	44.8	0.201438849	0.798561151	59616
75-79	83	0.3436853	0.6563147	47607
80-84	108.1	0.42550679	0.57449321	31245
85+	169	1	0	17950

CREATING ABRIDGED PERIOD LIFE TABLES: 4

- **Number Surviving at exact ages x**

$$l_{x+n} = l_x \times {}_np_x$$

We assume an initial population for our synthetic cohort : ***radix, lets say 100,000***

Then we keep on calculating l_x for each successive age group

REMEMBER: for calculations , ${}_np_x$ refers to

*probability of survival of **previous** period*

Can be interpreted as number entering any age group

Age Group	${}_nM_x$	${}_nq_x$	${}_np_x$	l_x
< 1	38.7	0.03796537	0.96203463	100000
1-4	1.4	0.005584364	0.994415636	96203
5-9	0.6	0.002995507	0.997004493	95666
10-14	0.5	0.002496879	0.997503121	95380
15-19	1.2	0.005982054	0.994017946	95142
20-24	1.6	0.007968127	0.992031873	94572
25-29	1.3	0.006478943	0.993521057	93819
30-34				
35-39				
40-44				
45-49				
50-54				
55-59	13.6	0.065764023	0.934235977	82216
60-64	18.8	0.089780325	0.910219675	76810
65-69	31.8	0.147290412	0.852709588	69914
70-74	44.8	0.201438849	0.798561151	59616
75-79	83	0.3436853	0.6563147	47607
80-84	108.1	0.42550679	0.57449321	31245
85+	169	1	0	17950

76810 persons have
(a) entered 60-64 yr age group
(b) Celebrated 60th birthday
(c) Survived 55-59 group

CREATING ABRIDGED PERIOD LIFE TABLES: 5

- **Deaths between at exact ages**

$${}_{n}d_{x} = l_{x} \times {}_{n}q_{x}$$

number entering the age-group **X**
probability of dying in the age group

In last age group, all will die

*REMEMBER: for any calculation , ${}_{n}q_{x}$ refers to probability of death of **the same** period*

Age Group	${}_{n}M_{x}$	${}_{n}q_{x}$	${}_{n}p_{x}$	l_{x}	${}_{n}d_{x}$
< 1	38.7	0.03796537	0.96203463	100000	3797
1-4	1.4	0.005584364	0.994415636	96203	537
5-9	0.6	0.002995507	0.997004493	95666	287
10-14	0.5	0.002496879	0.997503121	95380	238
15-19	1.2	0.005982054	0.994017946	95142	569
20-24	1.6	0.007968127	0.992031873	94572	754
25-29	1.3	0.006478943	0.993521057	93819	608
30-34	2.9	0.014395632	0.985604368	93211	1342
35-39	3.4	0.016856718	0.983143282	91869	1549
40-44	3.6	0.017839445	0.982160555	90321	1611
45-49	7.1	0.034880865	0.965119135	88709	3094
50-54	8.1	0.039696153	0.960303847	85615	3399
55-59	13.6	0.065764023	0.934235977	82216	5407
60-64	18.8	0.089780325	0.910219675	76810	6896
65-69	31.8	0.147290412	0.852709588	69914	10298
70-74	44.8	0.201438849	0.798561151	59616	12009
75-79	83	0.3436853	0.6563147	47607	16362
80-84	108.1	0.42550679	0.57449321	31245	13295
85+	169	1	0	17950	17950

CREATING ABRIDGED PERIOD LIFE TABLES: 6

- **Average Number living over the period n between ages x , $x+n$**

(Capital L) $_nL_x = n \times \frac{1}{2} (l_x + lx_{+n})$

Take the average of persons entering the current age group and the persons entering the *next* age group. (assumes uniform distribution of mortality)

Approximately these many people will be alive in each single year cohort and there will be n such cohorts

Eg $_4L_1 = 4 \times \frac{1}{2} (l_1 + l_5)$

In first year of life risk of death is higher in infants, so $_1L_0 = (0.3\, l_0 + 0.7\, l_1)$

In Last Group $_\infty L_{85} = \frac{l_{85}}{_\infty M_{85}}$ (where M is expressed as a fraction *derivaton*)

Age Group	$_nM_x$	$_nq_x$	$_np_x$	l_x	$_nd_x$	$_nL_x$
< 1	38.7	0.03796537	0.96203463	100000	3797	97342
1-4	1.4	0.005584364	0.994415636	96203	537	383739
5-9	0.6	0.002995507	0.997004493	95666	287	477615
10-14	0.5	0.002496879	0.997503121	95380	238	476303
15-19	1.2	0.005982054	0.994017946	95142	569	474285
20-24	1.6	0.007968127	0.992031873	94572	754	470978
25-29	1.3	0.006478943	0.993521057	93819	608	467574
30-34	2.9	0.014395632	0.985604368	93211	1342	462700
35-39	3.4	0.016856718	0.983143282	91869	1549	455474
40-44	3.6	0.017839445	0.982160555	90321	1611	447574
45-49	7.1	0.034880865	0.965119135	88709	3094	435811
50-54	8.1	0.039696153	0.960303847	85615	3399	419578
55-59	13.6	0.065764023	0.934235977	82216	5407	397565
60-64	18.8	0.089780325	0.910219675	76810	6896	366808
65-69	31.8	0.147290412	0.852709588	69914	10298	323824
70-74	44.8	0.201438849	0.798561151	59616	12009	268057
75-79	83	0.3436853	0.6563147	47607	16362	197130
80-84	108.1	0.42550679	0.57449321	31245	13295	122988
85+	169	1	0	17950	17950	106214

$_nL_x$ can be used to create population pyramids

If we do a survey in the population where 100,000 births are occurring each year, then we will find $_nL_x$ proportions of people in each age group

CREATING ABRIDGED PERIOD LIFE TABLES: 7

- **Total Population Aged x and over: T_x**

(Capital T) $T_x = \sum_{i=x}^{\infty} {}_nL_x$

Eg Total population aged 65 and over is given by

$T_{65} = {}_5L_{65} + {}_5L_{70} + {}_5L_{75} \ldots \ldots + {}_\infty L_{85}$

Interpretation: The number of life years that will be lived by people who are exactly 65 years of age

For last age group, $T_{85} = {}_\infty L_{85}$

Age Group	$_nM_x$	$_nq_x$	$_np_x$	l_x	$_nd_x$	$_nL_x$	T_x
< 1	38.7	0.03796537	0.96203463	100000	3797	97342	6851560
1-4	1.4	0.005584364	0.994415636	96203	537	383739	6754218
5-9	0.6	0.002995507	0.997004493	95666	287	477615	6370478
10-14	0.5	0.002496879	0.997503121	95380	238	476303	5892863
15-19	1.2	0.005982054	0.994017946	95142	569	474285	5416561
20-24	1.6	0.007968127	0.992031873	94572	754	470978	4942276
25-29	1.3	0.006478943	0.993521057	93819	608	467574	4471298
30-34	2.9	0.014395632	0.985604368	93211	1342	462700	4003724
35-39	3.4	0.016856718	0.983143282	91869	1549	455474	3541023
40-44	3.6	0.017839445	0.982160555	90321	1611	447574	3085549
45-49	7.1	0.034880865	0.965119135	88709	3094	435811	2637975
50-54	8.1	0.039696153	0.960303847	85615	3399	419578	2202164
55-59	13.6	0.065764023	0.934235977	82216	5407	397565	1782586
60-64	18.8	0.089780325	0.910219675	76810	6896	366808	1385021
65-69	31.8	0.147290412	0.852709588	69914	10298	323824	1018213
70-74	44.8	0.201438849	0.798561151	59616	12009	268057	694390
75-79	83	0.3436853	0.6563147	47607	16362	197130	426332
80-84	108.1	0.42550679	0.57449321	31245	13295	122988	229202
85+	169	1	0	17950	17950	106214	106214

T_x tells us the number of years people in each age group will live

Eg. The 100,000 infants just born still have 6,851,560 years of life ahead of them

CREATING ABRIDGED PERIOD LIFE TABLES: 8

- **Life Expectancy at age x: Average number of years lived by a person aged x = e_x**

The number of life years that will be lived by people who are exactly x years of age divided by the number of people who are aged x year

$$e_x = \frac{T_x}{l_x}$$

Eg. $e_1 = \frac{T_1}{l_1}$

Can be calculated for each age

Age Group	${}_nM_x$	${}_nq_x$	${}_np_x$	l_x	${}_nd_x$	${}_nL_x$	T_x	e_x
< 1	38.7	0.03796537	0.96203463	100000	3797	97342	6851560	68.5
1-4	1.4	0.005584364	0.994415636	96203	537	383739	6754218	70.2
5-9	0.6	0.002995507	0.997004493	95666	287	477615	6370478	66.6
10-14	0.5	0.002496879	0.997503121	95380	238	476303	5892863	61.8
15-19	1.2	0.005982054	0.994017946	95142	569	474285	5416561	56.9
20-24	1.6	0.007968127	0.992031873	94572	754	470978	4942276	52.3
25-29	1.3	0.006478943	0.993521057	93819	608	467574	4471298	47.7
30-34	2.9	0.014395632	0.985604368	93211	1342	462700	4003724	43.0
35-39	3.4	0.016856718	0.983143282	91869	1549	455474	3541023	38.5
40-44	3.6	0.017839445	0.982160555	90321	1611	447574	3085549	34.2
45-49	7.1	0.034880865	0.965119135	88709	3094	435811	2637975	29.7
50-54	8.1	0.039696153	0.960303847	85615	3399	419578	2202164	25.7
55-59	13.6	0.065764023	0.934235977	82216	5407	397565	1782586	21.7
60-64	18.8	0.089780325	0.910219675	76810	6896	366808	1385021	18.0
65-69	31.8	0.147290412	0.852709588	69914	10298	323824	1018213	14.6
70-74	44.8	0.201438849	0.798561151	59616	12009	268057	694390	11.6
75-79	83	0.3436853	0.6563147	47607	16362	197130	426332	9.0
80-84	108.1	0.42550679	0.57449321	31245	13295	122988	229202	7.3
85+	169	1	0	17950	17950	106214	106214	5.9

e_x tells us the average life years that will be lived by those who are x year of age

Here:

e_0 = 68.5 year
e_1 = 70.2 year

Can you explain, Why $e_1 > e_0$

USING PERIOD LIFE TABLES

- If we have a cohort of children who follow the observed mortality patterns, what would be
  - Life expectancies
  - Total person-years lived
  - Proportion in various age groups at any given point of time

- More complex life tables account for
  - Gender differentials in mortality
  - Migrations
  - Fertility patterns etc

- Model life tables are routinely prepared by census agencies:
  - Eg. SRS Model Life Tables

- Period life expectancies are a useful measure of mortality rates actually experienced over a given period and, for past years, provide an objective means of comparison of the trends in mortality over time, between areas of a country and with other countries.
- Period life expectancies are sometimes mistakenly interpreted by users as allowing for subsequent mortality changes. Period life expectancy answers the question ‘For a group of people aged x in a given year, how long would they live, on average, if they experienced the age-specific mortality rates above age x of the period in question over the course of their remaining lives?’

- Cohort life expectancies, even for past years, usually require projected mortality rates for their calculation and hence, in such cases, involve an element of subjectivity.
- The cohort life expectancy answers the question ‘For a group of people aged x in a given year, how long would we expect them to live, on average, if they experienced the actual or projected future age-specific mortality rates not from the given year but from the series of future years in which they will actually reach each succeeding age if they survive?’ If mortality rates at age x and above are projected to decrease in future years, the cohort life expectancy at age x will be greater than the period life expectancy at age x.

BEYOND SIMPLE…LIFE CAN BE VERY COMPLICATED

STABLE POPULATION

- Will grow
- Will *probably* not alter its age-sex structure much

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DERIVATION OF FORMULA OF q_x FOR SINGLE YEAR

Probability of death = observed deaths among persons aged x on last birthday / *initial* population with age x

We know the mid-year population but do not know *initial* population.

Assuming death events are evenly spread over the year, then the initial population = mid year population + ½ annual deaths

$$q_x = \frac{Dx}{Px + 0.5\,Dx}$$

Where: Dx = observed deaths among persons aged x on last birthday

Px = Observed **Mid-year** population

However we do not know the Dx and Px. But we know M_x and $M_x = Dx/Px$

Divide both numerator and denominator by Px

$$q_x = \frac{Dx/Px}{1 + 0.5\,Dx/Px} = \frac{Mx}{1 + 0.5\,Mx} = \frac{2Mx}{2 + Mx}$$

DERIVATION OF FORMULA OF ${}_nq_x$ FOR GROUPED DATA

Probability of death needs to be calculated over n Years for age-group $(x,\ x+n)$

1. We need to calculate the total number of deaths for age-group $(x,\ x+n)$ that will occur over the n years (since we need probability over n years)

2. We need to calculate the Initial population for age-group $(x,\ x+n)$

1. Knowing that ${}_nD_x$ deaths occur in 1 year in the age-group, then total number of deaths over n years in the age-group = $n \times {}_nD_x$

2. Assuming that mid-year population approximates the average population over n years and that the death events are evenly spread over n years, then the initial population of age group = mid year population + ½ total deaths over n years

$$ {}_nq_x = \frac{n \times nDx}{nPx + n \times 0.5\, nDx} = \frac{2n \times nMx}{2 + n \times nMx} $$

Where: nDx = Observed deaths for age-group $(x,\ x+n)$ in 1 year

nPx = Observed **Mid-year** population for age-group $(x,\ x+n)$, AND

nMx = Age Specific death rates for age-group $(x,\ x+n)$

DERIVATION OF FORMULA OF $_{\infty}L_{85}$ FOR LAST GROUP

$$_{\infty}m_{85} = \frac{_{\infty}d_{85}}{_{\infty}L_{85}} \ggg \ _{\infty}L_{85} = \frac{_{\infty}d_{85}}{_{\infty}m_{85}}$$

Where $_{\infty}m_{85}$ is the calculated age specific death rate from life table death

Now assuming, $_{\infty}m_{85}$ is approximately equal to $_{\infty}M_{85}$

And for last age group, everyone who enterd the age group will die and thereby $_{\infty}d_{85} = l_{85}$

- Therefore,

$$_{\infty}L_{85} = \frac{l_{85}}{_{\infty}M_{85}}$$

DERIVATION OF FORMULA OF ${}_nq_x$ FOR GROUPED DATA